ST. HENRY'S COLLEGE KITOVU

A'LEVEL PURE MATHEMATICS P425/1 SEMINAR QUESTIONS 2019

ALGEBRA

1. Solve the simultaneous equations:
   (a) $x^2 + y^2 = 5$, $\dfrac{1}{x^2} + \dfrac{1}{y^2} = \dfrac{5}{4}$
   (b) $\dfrac{x}{y} + \dfrac{y}{x} = \dfrac{17}{4}$, $x^2 - 4xy + y^2 = 1$

2. Find the range of values of x for which
   (a) $\dfrac{2x+1}{x+2} > \dfrac{1}{2}$.
   (b) $|2x+1| > 7$

3. Resolve into partial fractions
   (a) $\dfrac{x^3 + x^2 + 4x}{x^2 + x - 2}$
   (b) $\dfrac{3x^2 + 8x + 13}{(x-1)(x^2 + 2x + 5)}$
   (c) $\dfrac{2x^3 + 2x^2 + 2}{(x+1)^2(x^2+1)}$

4. Solve the following equations:
   (a) $2^{3x+1} = 5^{x+1}$
   (b) $9^x - 4(3^x) + 3 = 0$
   (c) $\log_x 9 + \log_{x^2} 3 = 2.5$
   (d) $\sqrt{2x-1} - \sqrt{x-1} = 1$
   (e) $\begin{aligned} 2x + 3y + 4z &= 8 \\ 3x - 2y - 3z &= -2 \\ 5x + 4y + 2z &= 3 \end{aligned}$

5. Find:
   (a) The three numbers in arithmetic progression such that their sum is 27 and their product is 504
   (b) The three numbers in a geometrical progression such that their sum 39 and their product is 729.
   (c) The sum of the last three terms of a geometrical progression having n terms is 1024 times the sum of the first three terms of the progression. If the third term is 5, find the last term.

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(d) Prove by induction that $1^3+2^3+....+n^3=\frac{1}{4}n^2(n+1)^2$ and deduce that

$1^3+3^3+5^3....+(2n+1)^3=(n+1)^2(2n^2+4n+1)$

6. Expand:

(a) $\frac{7+x}{(1+x)(1+x^2)}$ in ascending powers of x as far as the term in x^4.

(b) $\left(1-\frac{3}{2}x-x^2\right)^5$ in ascending powers of x as far as the term in x^4.

(c) Find the term independent of x in the expansion of $\left(2x+{}^1\!/_{x^2}\right)^{12}$ in descending powers of x and find the greatest term in the expansion when $x=\frac{2}{3}$.

(d) Find by binomial theorem, the coefficient of x^8 in the expansion $(3-5x^2)^{1/2}$ in ascending powers of x.

(e) In the binomial expansion of $(1+x)^{n+1}$, n being an integer greater than two, the coefficient of x^4 is six times the coefficient of x^2 in the expansion $(1+x)^{n-1}$. Determine the value of n.

7. (a) Without using the calculator, simplify $\frac{\left(\cos\left(\frac{\pi}{9}\right)+i\sin\left(\frac{\pi}{9}\right)\right)^4}{\left(\cos\left(\frac{\pi}{9}\right)-i\sin\left(\frac{\pi}{9}\right)\right)^5}$

(b) In a quadratic equation $z^2+(p+iq)z+3i=0$. p and q are real constants. Given that the sum of the squares of the roots is 8. Find all possible pairs of values of p and q.

8. (a) How many different arrangements of letters can be made by using all the letters in the word contact? In how many of these arrangements are the vowels separated?
(b) In how many ways can a team of eleven be picked from fifteen possible players.

9. (a) If α and β are the roots of the equation $x^2-px+q=0$, form the equation whose roots are $\frac{\alpha}{\beta^2}$ and $\frac{\beta}{\alpha^2}$.

(b) If α and β are the roots of the equation $x^2+bx+c=0$, form the equation whose roots are $\frac{1}{\beta^3}$ and $\frac{1}{\alpha^3}$. If in the equation above $\alpha\beta^2=1$, prove that $a^3+c^3+abc=0$

10. (a) If $z=x+iy$ and $\bar{z}$ is the conjugate of z, find the values of x and y such that $\frac{1}{z}+\frac{2}{\bar{z}}=1+i$

(b) If x, y, a and b are real numbers and if $x+iy=\frac{a}{b+\cos\theta+i\sin\theta}$. Show that

$(b^2-1)(x^2+y^2)+a^2=2abx$.

(c) If n is an integer and $z=cos\theta+sin\theta$, show that $2\cos n\theta=z^n+\frac{1}{z^n}$, $2i\sin n\theta=z^n-\frac{1}{z^n}$.

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Use the result to establish the formula $8\cos^4\theta = \cos 4\theta + 4\cos 2\theta + 3$.

(e) If z is a complex number and $\left|\frac{z-1}{z+1}\right| = 2$, find the equation of the curve in the Argand diagram on which the point representing z lies.

TRIGONOMETRY

11. If $\sin\theta + \sin\beta = a$ and $\cos\theta + \cos\beta = b$, show that $\cos^2\left(\frac{\theta-\beta}{2}\right) = \frac{1}{4}\left(a^2+b^2\right)$

12. Show that $\sin 7x + \sin x - 2\sin 2x\cos 3x = 4\cos^3 3x$

13. If A, B and C are angles of a triangle, show that:

(i) $\cos A + \cos(B-C) = 2\sin B\sin C$

(ii) $\cos\frac{C}{2} + \sin\frac{A-B}{2} = 2\sin\frac{A}{2}\cos\frac{B}{2}$

14. Express $y = 8\cos x + 6\sin x$ in form of $R\cos(x-\alpha)$ where R is positive and α is acute. Hence find the maximum and minimum values of $\frac{1}{8\cos x + 6\sin x + 15}$ and the corresponding angle respectively.

15. Show that:

(a) $\tan^{-1} x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$

(b) $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

(c) Find x if $\tan^{-1} x + \tan^{-1}(1-x) = \tan^{-1}\left(\frac{4}{3}\right)$

16. (a) Show that $\cos 4\theta = \frac{\tan^4\theta - 6\tan^2\theta + 1}{\tan^4\theta + 2\tan^2\theta + 1}$

(b) Solve the equation $8\cos^4 x - 10\cos^2 x + 2 = 0$ for x in the range of $0° \le x \le 180°$

17. (a) If $\tan\theta = \frac{1}{p}$ and $\tan\beta = \frac{1}{q}$ and $pq = 2\text{p}$, show that $\tan(\theta+\beta) = p+q$

(b) Show that $\sin 2A + \cos 2A = \frac{(1+\tan A)^2 - 2\tan^2 A}{1+\tan^2 A}$

18. If α, β and γ are all greater than $\frac{\pi}{2}$ and less than 2π and $\sin\alpha = \frac{1}{2}$, $\tan\beta = \sqrt{3}$, $\cos\gamma = \frac{1}{\sqrt{2}}$.

Find the value of $\tan(\alpha+\beta+\gamma)$ in surd form.

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19. Solve for x in the range $0°$ to $360°$

(a) $3\cos^2 x - 3\sin x\cos x + 2\sin^2 x = 1$

(b) $4\cos x = 3\tan x + 3\sec x$

20. Prove that $4\cos\theta\cos 3\theta + 1 = \frac{\sin 5\theta}{\sin\theta}$. Hence find all the values of θ in the range $0°$ to $180°$ for which $\cos\theta\cos 3\theta = \frac{-1}{2}$

VECTORS

21. The coordinates of the points A and B are (0,2,5) and (-1,3,1) and the equation of the line L is $\frac{x-3}{2} = \frac{y-2}{-2} = \frac{z-2}{-1}$

(i) Find the equation of the plane containing the point A and perpendicular to L and verify that B lies in the plane.

(ii) Show that the point C in which L meets the plane is (1,4,3) and find the angle between CA and CB

22. (a) A body moves such that its position is given by $OP = (3sint)i + (3cost)j$ where O is the origin and t is the time. Prove that the velocity of the particle when at P is perpendicular to OP.

(b) The lines L_1 and L_2 have Cartesian equations $\frac{x}{1} = \frac{y+2}{2} = \frac{z-5}{-1}$ and $\frac{x-1}{-1} = \frac{y+3}{-3} = \frac{z-6}{1}$.

Show that L_1 and L_2 intersect and find the coordinates of the point of intersection.

23. (a) Find the acute angle between the lines whose equations are $\frac{x-2}{-4} = \frac{y-3}{3} = \frac{z+1}{-1}$ and $\frac{x-3}{2} = \frac{y-1}{6} = \frac{z+1}{-5}$.

(b) The points A and B have coordinates (1,2,3) and (4,6,-2) respectively and the plane has equation $x + y - z = 24$. Determine the equation of the line AB, hence the angle this line makes with the plane.

24. (a) Find the perpendicular distance of the line $\frac{x-5}{1} = \frac{y-6}{2} = \frac{z-3}{4}$ from the point (-6,-4,-5).

(b) Find the shortest distance between the two skew lines $\frac{x+1}{1} = \frac{y-2}{2} = \frac{z-3}{1}$ and $\frac{x}{2} = \frac{y+1}{1} = \frac{z-1}{3}$ respectively.

(c) Find the perpendicular distance of the plane $2x - 14z + 5z = 10$ from the origin.

25. (a) Show that the line $\frac{x-2}{2} = \frac{y-2}{-1} = \frac{z-3}{3}$ is parallel to the plane $4x - y - 3z = 4$ and find the perpendicular distance from the line to the plane.

(b) Find the Cartesian equation of the line of intersection of the two planes $2x - 3y - z = 1$ and $3x + 4y + 2z = 3$.

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26. (a) Find the Cartesian equation of the plane containing the point (1,3,1) and parallel to the vectors $\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$

(b) Find the Cartesian equation of the plane containing the points (1,2,-1), (2,1,2) and (3,-3,3).

27. Given the points A, B and C with coordinates (2,5,-1), (3,-4,2) and C(-1,2,1). Show that ABC is a triangle and find the area of the triangle ABC

28. (a) Find the angle between the parallel planes $3x + 2y - z = -4$ and $6x + 4y - 2z = 6$.

(b) Find the acute angle between the planes $2x + y + 3z = 5$ and $2x + 3y + z = 7$

29. The points A and B have coordinates (2,1,1) and (0,5,3) respectively. Find the equation of the line AB. If C is the point (5,-4,2). Find the coordinates of D on AB such that CD is perpendicular to AB. Find the equation of the plane containing AB and perpendicular to the line CD.

30. (a) Given that $OP = \begin{pmatrix} 4 \\ -3 \\ 5 \end{pmatrix}$ and $OQ = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$, find the coordinates of the point R such that $\overline{PR} = \overline{PQ} = 1:2$ and the points P, Q and R are collinear.

(b) A and B are the points (3,1,1) and (5,2,3) respectively, and C is a point on the line $\boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$. If angle BAC=$90^0$, find the coordinates of C

ANALYSIS

31. Differentiate from first principles

(a) $y = \tan^{-1} x$

(b) $y = ax^n$

(c) $y = \sin 3x$

32. Find the derivative of:

(a) $y = 5\sin^{-1}(4x)$

(b) $y = \tan^{-1}\left(\dfrac{1+\tan x}{1-\tan x}\right)$

(c) $y = \dfrac{\sin x}{x^2 + \cos x}$

(d) $y = \sqrt{\dfrac{x}{1+x}}$

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33. Find:

(a) $\int \sin^{-1} x$

(b) $\int \frac{dx}{x^2+4x+13}$

(c) $\int \frac{dx}{x\log_e x}$

(d) $\int \frac{dx}{(1+x^2)\tan^{-1} x}$

(e) Show that $\int_0^2 \sqrt{\frac{x}{4-x}}dx = \pi - 2$

(f) Show that $\int_1^{10} x\log_{10} x = 50 - \frac{99}{4\ln 10}$

34. (a) If $x = t^3$ and $y = 2t^2$. Find $\frac{dy}{dx}$ in terms of t and show that when $\frac{dy}{dx} = 1$, $x = 2$ or $x = \frac{10}{27}$

(b) If $y = \frac{2t}{1+t^2}$ and $x = \frac{1-t^2}{1+t^2}$, find $\frac{d^2y}{dx^2}$ in terms of t

35. Given that:

(a) $y = \sqrt{4+3\sin x}$, show that $2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + y^2 = 4$

(b) $y = e^{2x}\cos 3x$, show that $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0$

(c) $y = \left(x+\sqrt{1+x^2}\right)^p$, show that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - p^2y = 0$

(d) $y = \sin(\log_e x)$, show that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

36. (a) Find the volume generated when the area enclosed by the curve $y = 4x - x^2$ and the line $y = 2x$ is rotated completely about the x – axis.

(b) Find the area contained between the two parabolas $4y = x^2$ and $4x = y^2$.

(c) Find the area between the curve $y = x^3$, the x – axis and the lines $y = 1, y = 8$.

(d) Find the area of the curve $x^2 + 3xy + 3y^2 = 1$

(e) Show that in the solid generated by the revolution of the rectangular hyperbola $x^2 - y^2 = a^2$ about the x – axis, the volume of the segment of height a from the vertex is $\frac{4}{3}\pi a^3$

37. (a) A right circular cone of semi – vertical angle θ is circumscribed about a sphere of radius R. show that the volume of the cone is $V = \frac{1}{3}\pi R^3(1+\operatorname{cosec}\theta)^3\tan^2\theta$ and find the value of θ when the volume is minimum.

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(b) Water is poured into a vessel, in the shape of a right circular cone of vertical angle 90°, with the axis vertical, at the rate of $125\text{cm}^3/s$. At what rate is the water surface rising when the depth of the water is 10cm?

38. Sketch the curve $y = \frac{x}{x+2}$. Find the area enclosed by the curve, the lines $x = 0, x = 1$ and the line $y = 1$. Also find the volume generated when this area revolves through 2π radians about the line $y = 1$.

39. Solve the differential equations below:

(a) $\frac{1}{3x}\frac{dy}{dx} + \cos^2 y = 1$, when $x = 2$ and $y = \frac{\pi}{4}$

(b) $(x-y)\frac{dy}{dx} = x + y$, when $x = 4$ and $y = \pi$

(c) $\frac{dy}{dx} + 3y = e^{2x}$, when $x = 0$ and $y = \frac{6}{5}$

40. In a certain type of chemical reaction a substance A is continuously transformed into a substance B. throughout the reaction, the sum of the masses of A and B remains constant and equal to m. The mass of B present at time t after the commencement of the reaction is denoted by x. At any instant, the rate of increase of mass of B is k times the mass of A where k is a positive constant.

(a) Write down a differential equation relating x and t

(b) Solve this differential equation given that $x = 0$ and $t = 0$. Given also that $x = \frac{1}{2}m$ when $t = \ln 2$, determine the value of k and show that at time t, $x = m(1 - e^{-t})$. Hence find:

(i) The value of x (in terms of m) when $t = 3\ln 2$

(ii) The value of t when $x = \frac{3}{4}m$

GEOMETRY

41. (a) Find the equation of a line which makes an angle of 150° with the x – axis and y – intercept of -3 units.

(b) Find the acute angle between the lines $3y - x = 4$ and $6y - 3x - 5 = 0$

(c) OA and OB are equal sides of an isosceles triangle lying in the first quadrant. OA and OB make angles θ_1 and θ_2 with x – axis respectively. Show that the gradient of the bisector of the acute angle AOB is $\operatorname{cosec}\theta - \cot\theta$ where $\theta = \theta_1 + \theta_2$

(d) Find the length of the perpendicular from the point P(2,-4) to the line $3x + 2y - 5 = 0$

42. (a) Find the equation of the circle with centre (4,-7) which touches the line $3x + 4y - 9 = 0$

(b) Find the equation of the circle through the points (6,1), (3,2), (2,3)

(c) Find the equation of the circumcircle of the triangle formed by three lines $2y - 9x + 26 = 0$, $9y + 2x + 32 = 0$ and $11y - 7x - 27 = 0$

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43. (a) Find the length of the tangent from the point (5,6) to the circle $x^2+y^2+2x+4y-21=0$.

(b) Find the equations of the tangents to the circle $x^2+y^2=289$ which are parallel to the line $8x-15y=0$

(c) Find the equation of the circle of radius $12\tfrac{4}{5}$ which touches both the lines $4x-3y=0$ and $3x+4y-13=0$ and intersects the positive y – axis.

(d) A circle touches both the x – axis and the line $4x-3y+4=0$. Its centre is in the first quadrant and lies on the line $x-y-1=0$. Prove that its equation is $x^2+y^2-6x-4y+9=0$

44. Find the equations of the parabolas with the following foci and directrices:

(i) Focus (2,1), directrix $x=-3$

(ii) Focus (0,0), directrix $x+y=4$

(iii) Focus (-2,-3), directrix $3x+4y-3=0$

45. (a) Show that the curve $x=5-6y+y^2$ represents a parabola. Find its focus and directrix, hence sketch it.

(b) Find the equation of the normal to the curve $y^2=4bx$ at the point $P(bp^2,2bp)$. Given that the normal meets the curve again at $Q(bq^2,2bq)$, prove that $p^2+pq+2=0$

46. (a) Show that the equation of the normal with gradient m to the parabola $y^2=4ax$ is given by $y=mx-2am-am^3$.

(b) P and Q are two points on the parabola $y^2=4ax$ whose coordinates are $P(ap^2,2ap)$ and $Q(aq^2,2aq)$ respectively. If OP is perpendicular to OQ, show that $pq=-4$ and that the tangents to the curve at P and Q meet on the line $x+4a=0$

47. (a) A conic is given by $x=4\cos\theta$, $y=3\sin\theta$. Show that the conic is an ellipse and determine its eccentricity

(b) Given that the line $y=mx+c$ is a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, show that $c^2=a^2m^2+b^2$. Hence determine the equations of the tangents at the point (-3,3) to the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$.

48. (a) Show that the locus of the point of intersection of the tangents to an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ which are at right angles to one another is a circle $x^2+y^2=a^2+b^2$.

(b) The normal to the ellipse $x^2+y^2=100$ at the points A(6,4) and B(8,3) meet at N. If P is the mid – point of AB and O is the origin, show that OP is perpendicular to ON.

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49. (a) P is a point $\left(ap^2, 2ap\right)$ and Q the point $\left(aq^2, 2aq\right)$ on the parabola $y^2 = 4ax$. The tangents at P and Q intersect at R. Show that the area of triangle PQR is $\frac{1}{2}a^2\left(p-q\right)^3$

(b) The normal to the parabola $y^2 = 4ax$ at $P\left(ap^2, 2ap\right)$ meets the axis of the parabola at M and MP is produced beyond P to Q so that MP = PQ. Show that the locus of Q is $y^2 = 16a\left(x+2a\right)$

50. (a) The normal to the rectangular hyperbola $xy = 8$ at the point (4,2) meets the asymptotes at M and N. Find the length of MN

(b) The tangent at P to the rectangular hyperbola $xy = c^2$ meets the lines $x - y = 0$ and $x + y = 0$ at A and B and Δ denotes the area of triangle OAB where O is the origin. The normal at P meets the x – axis at C and the y – axis at D. if Δ_1 denotes the area of the triangle ODC. Show that $\Delta^2\Delta_1 = 8c^6$

END

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S.5 MATHEMATCIS PAPER ONE

CHAPTER ONE : DIFFERENTIATION

REVISION QUESTIONS 2020

Attempt all questions

1. Find the gradient function $\frac{dy}{dx}$ for each of the following functions.

   a) $y = x^2 + 7x - 4$

   b) $y = 6x^2 - 7x + 8$

   c) $y = 3x^6 - 7x^2 + 6x - 8$

   d) $y = 3x - \frac{5}{x} + \frac{6}{x^2}$

   e) $(x^7+2)(4x-1)$

2. Find the gradients of the following lines at the points indicated.

   a) $y=2x^3-x^2 + 3x - 1$ at $(-1,-6)$

   b) $y = x^2 + 7x - 4$ at $(2,21)$

   c) $y = 2x^2 - x + \frac{4}{x}$ at $(2,8)$

   d) $y = 3x + \frac{1}{x}$ at $(1,4)$

   e) $y = (x+1)(2x + 3)$ at $(2,21)$

3. If $f(x) = x^3 + 4x$ find

   a) $f(1)$

   b) $f'(x)$

   c) $f'(1)$

   d) $f''(x)$

e) $f''(2)$

4. If $f(x) = 3x^2 + \frac{24}{x}$ find

   a) $f'(x)$

   b) $f'(-12)$

5. Find the equation of the normal to the curve $y=x^2 + 4x-3$ at the point where the curve cuts the y- axis. Ans : $4y + x + 12 = 0$

6. Find the equation of the tangent to the curve $y=x^2-3x-4$ at the point where this curve cuts the line $x = 5$. Ans: $y = 7x-29$

7. Find the equation of the tangent to the curve $y = (2x-3)(x-1)$ at each of the points where this curve cuts the x- axis. Find the point of intersection of these tangents.

   Ans: $y + x = 1$, $2y = 2x-3;(\frac{5}{4}, -\frac{1}{4})$

8. Find the equation of the normal to the curve $y=x^2-6x + 5$ at each of the points where the curve cuts the x- axis. Ans: $4y-x + 1 = 0$, $4y + x-5 = 0$

9. Find the equation of the tangent to the curve $y=x^2 + 5x-3$ at the points where the line $y = x + 2$ crosses the curve. Ans: $y = 7x-4, y + 5x + 28 = 0$

10. Find the coordinates of the point on the curve $y=2x^2$ at which the gradient is 8 Hence find the equation of the tangent to $y=2x^2$ whose gradient is 8. Ans: (2 ,8),$y = 8x-8$

11. Find the coordinates of the point on the curve $3x^2-1$ at which the gradient is 3. Ans: $(\frac{1}{2}, -\frac{1}{4})$

12. Find the equation of the tangent to the curve $y = 2x^2-2x + 1$ which has a gradient of 0.5

    Ans :$2y = x + 2$

13. Find the value of k for which $y = 2x + k$ is a tangent to the curve $y = 2x^2-3$. Ans $k = -\frac{7}{2}$

14. Find the equation of the tangent to the curve $y = (x-5)(2x + 1)$ which is parallel to the x- axis. Ans: $8y + 121 = 0$

15. A curve has the equation $y = x^3-px + q$. The tangent to the curve at the point (2,-8) is parallel to the x-axis. Find the values of p and q. find also the coordinates of the other point where the tangent is parallel to the x-axis. Ans p=12,q=8;(-2,24)

16. The function $ax^2 + bx + c$ has a gradient function $4x + 2$ and a stationary value of 1. Find the values of a, b and c. ***Ans a=2 , b=2 and c=$\frac{3}{2}$***

17. Find the second differential of y with respect to x for each of the following :

a) $y = 6x^2 + 7$

b) $y = 5x^3 + 6x - 5$

c) $y = 2 + \frac{3}{x}$

18. If $y = 3x^2 - x$ show that $y\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y + 1 = 6x$.

19. The tangent to the curve $y = ax^2 + bx + 2$ at $(1, \frac{1}{2})$ is parallel to the normal to the curve $y = x^2 + 6x + 10$ at (-2, 2). Find the values of a and b. Ans: 1 ,-2.5

20. Find the coordinates of any stationary points on the given curves and distinguish between them.

a) $y = 2x^2 - 8x$

b) $y = x^3 - x^2 - x + 7$

c) $y = 1 - 3x + x^3$

d) $y = (x-1)(x^2 - 6x + 2)$

e) $y = 18x - 20 - 3x^3$

f) $y = x^3 + 6x^2 + 12x + 12$

g) $y = x^3 - 3x^2 + 3x - 1$

21. Find the coordinates of the stationary points on the following curves and distinguish between them. Hence sketch the curves.

a) $y = x^4 + 2x^3$

b) $y = x^3 - 4x^2 + 4x$

c) $y = 5x^6 - 12x^5$

d) $y = x^4 - 4x^3 + 4x^2$

22. Differentiate $x^2 + \frac{1}{x}$ from first principles.

23. Differentiate $y = \frac{x}{x^2+1}$ with respect to x from first principles

24. Find the derivative of $\frac{1}{\sqrt{x}}$ from first principles

25. Find the equation of the normal to the curve $y = x^2 + 5x + 3$ that is parallel to the line

    $y = 9x$.

26. Differentiate $P = x - x^2 + \frac{\pi}{2x}$ with respect to x where π is a constant

27. Find the equation of a tangent to the curve $y=2-4^{x2}+^{x3}$ at a point (1,-1)

28. Find the stationary points of the curve $y= 5+24x-9x^2-2x^3$ and distinguish the nature of these stationary points.

29. P and Q are neighboring points on the curve $y = 2(x-x^2)$. P is the point (x,y) and Q the point $(x + \delta x, y + \delta y)$. Find the value of the ratio $\frac{\delta y}{\delta x}$ and determine the gradient of the curve at point P.

30. Differentiate the following using the first principles.

    a) $y = x^3 + x^2$

    b) $y = \frac{1}{x^2}$

    c) $y = \frac{1}{2x^2}$

31. P is the point (x,y) and Q the point $(x + \delta x, y + \delta y)$. On the graph of $y = \sqrt{x}$. Show that $\frac{\delta y}{\delta x} = \frac{1}{\sqrt{(x+\delta x)} + \sqrt{x}}$. And hence find the gradient of the curve at the point P.

32. Find the slope of the curve $y = ax^2 + bx + c$, where a,b and c are constants at the point whose x coordinate is x. At what point is the tangent to the curve parallel to the x- axis?

33. Find the gradient of the curve $y = 9x - x^3$ at the point where $x = 1$. Find the equation of the tangent to the curve at this point. Where does this tangent meet the line $y = x$?

34. Find the equation of the tangent at the point (2,4) to the curve $y = x^3 - 2x$. Also find the coordinates of the point where the tangent meets the curve again.

35. Find the equation of the tangent to the curve $y = x^3 - 9x^2 + 20x - 8$ at the point (1,4). At what points of the curve is the tangent parallel to the line $4x + y - 3 = 0$?

36. Find the equation of the tangent to the curve $y = x^3 + \frac{1}{2}x^2 + 1$ at the point $(-1, \frac{1}{2})$. Find the coordinates of another point on the curve where the tangent is parallel to that at the point $(-1, \frac{1}{2})$.

37. Find the points of intersection with the x-axis of the curve $y = x^3 - 3x^2 + 2x$,and find the equation of the tangent to the curve at each of these points.

38. Find the equations of the normals to the parabola $4y=x^2$ at the points (-2,1) and (-4,4). Show that the point of intersection of these two points lies on the parabola.

39. Find the equation of the tangent at the point (1,-1) to the curve $y = 2 - 4x^2 + x^3$. What are the coordinates of the point where the tangent meets the curve again? Find the equation of the tangent at this point.

40. Find the coordinates of the point P on the curve $8y=4-x^2$ at which the gradient is $\frac{1}{2}$. Write down the equation of the tangent to the curve at P. find also the equation of the tangent to the curve whose gradient is $-\frac{1}{2}$, and the coordinates of its point of intersection with the tangent at P.

41. Find the equations of the tangents to the curve $y = x^3 - 6x^2 + 12x + 2$ which are parallel to the line $y = 3x$.

42. Find the coordinates of the points of intersection of the line $x - 3y = 0$ with the curve $y = x(1-x^2)$. If these points are in order P,O,Q, prove that the tangents to the curve at P and Q are parallel, and that the tangent at O is perpendicular to them.

43. Find the equations of the tangent and the normal to the parabola $x^2 = 4y$ at the point (6,9). Also find the distance between the points where the tangent and the normal meet the y-axis.

44. The curve $y = (x-2)(x-4)(x-3)$ cuts the x-axis at the points P(2,0) , Q(3,0), R(4,0). Prove that the tangents at P and R are parallel. At what point does the normal to the curve at Q cut the y-axis?

45. Find the equation of the tangent at the point P(3,9) to the curve $y = x^3 - 6x^2 + 15x - 9$

If O is the origin and N is the foot of the perpendicular from P to the x-axis, prove that the tangent at P passes through the mid-point of ON. Find the coordinates of another point on the curve, the tangent at which is parallel to the tangent at the point (3,9).

46. A tangent to the parabola $x^2 = 16y$ is perpendicular to the line $x - 2y - 3 = 0$. Find the equation of this tangent and the coordinates of its point of contact.

47. Find the equation of the tangent to $y = x^2$ at the point (1,1) and of the tangent to $y = \frac{1}{6}x^3$ at the point $(2,\frac{4}{3})$. Show that these tangents are parallel, and find the distance between them.

48. The curve C is defined by $y = ax^2 + b$, where a and b are constants. Given that the gradient of the curve at the point (2,-2) is 3, find the values of a and b.

49. Given that the curve with equation $y = Ax^2 + Bx$ has gradient 7 at the point (6,8), find the values of the constants A and B.

50. A curve with equation $y = A\sqrt{x} + \frac{B}{\sqrt{x}}$, for constants A and B, passes through the point (1,6) with gradient -1. Find A and B.

51. Find the equation of the tangent, t, to the curve $y = x^2 + 5x + 2$, which is perpendicular to the line, l, with equation $3y + x = 5$.

END

Permutations and combinations

Permutation

A permutation is an ordered arrangement of a number of objects

Consider digits 1, 2 and 3; find the possible arrangements of the digits

123, 132, 321
231, 321, 321 } The total of six

This problem may also be solved as follows:

Given the three digits above, the first position can take up three digits, the second position can take up two digits and the third position can take up 1 digit only

1^{st} position	2^{nd} position	3^{rd} position
3	2	1

The total is thus 3 x 2 x 1 = 6

If the digits were four say 1, 2, 3, 4 the arrangement would be

1^{st}	2^{nd}	3^{rd}	4^{th}
4	3	2	1

The total is thus 4 x 3 x 2 x 1 = 24

In summary the number of ways of arranging n different items in a row is given by n(n – 1)(n – 2)(n -3) x ……….2 x 1 and can be expressed as n!

If the total number of books is 6

The total number of arrangements = 6!

= 6 x 5 x 4 x 3 x 2 x 1 = 720 ways

Example 1

Find the values of the following expression

(a) 5!
Solution
5!= 5 x 4 x 3 x 2 x 1 = 120

(b) $\frac{8!}{5!}$
Solution
$\frac{8!}{5!} = \frac{8\,x\,7\,x\,6\,x\,5!}{5!} = 336$

(c) $\frac{10!}{6!x\,5!\,x\,2!}$
Solution
$\frac{10!}{6!x\,5!\,x\,2!} = \frac{10\,x\,9\,x\,8\,x\,7\,x\,6!}{6!x\,5\,x\,4\,x\,3\,x\,2\,x\,1\,x\,2x\,1} = 21$

(d) Four different pens and 5 different books are to be arranged on a row. Find
(i) The number of possible arrangements of items
Solution
Total number of items = 4 + 5 = 9
Total number of arrangements = 9!
= 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1
= 362,880 ways
(ii) The number of possible arrangements if three of books must be kept together
Solution
The pens are taken to be one since they are to be kept together. So we consider total number of items to six.
The number of arrangements of six items = 6! = 6 x 5 x 4 x 3 x 2 x 1
= 720 ways
The arrangement of 4 pens = 4!
= 4 x 3 x 2 x 1 =24
Total number arrangements of all the items = 720 x 24 = 17, 280

Multiplication principle of permutation

If one operation can be performed independently in a different ways and the second in b different ways, then either of the two events can be performed in (a + b) ways

Example 2

There are 6 roads joining P to Q and 3 roads joining Q to R. Find how many possible routes are from P to R

From P to Q = 6 ways

From Q to R = 3 ways

Number of routes from P to R = 6 x 3 = 18

Example 3

Peter can eat either matooke, rice or posh on any of the seven days of the week. In how many ways can he arrange his meals in a week

Solution

For each of the 7 days, there are 3 choices

Total number of arrangements

= 3 x 3 x 3 x 3 x 3 x 3 x 3 = 3^7 = 2187 ways

Example 4

There are four routes from Nairobi to Mombasa. In how many different ways can a taxi go from Nairobi to Mombasa and returning if for returning:

(a) any of the route is taken
= 4 x 4 = 16 ways
(b) the same route is taken
= 4 x 1 = 4 ways
(c) the same route is not taken
= 4 x 3 = 12 ways

Example 5

David can arrange a set of items in 5 ways and John can arrange the same set of items in 3 ways. In how many ways can either David or John arrange the items?

Solution

Number of ways in which David arranges = 5

Number of ways in which John arranges = 3

Number of ways in which either David or John arrange the items = 5 + 3 = 8 ways

The number of permutation of r objects taken from n unlike objects

The permutation of n unlike objects taking r at a time is denoted by ^nP_r which is defined as

$^nP_r = \frac{n!}{(n-r)!}$, where r ≤ n.

In case r = n, we have ^nP_n which is interpreted as the number of arranging n chosen objects from n objects denoted by n!

$^nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n! => 0! = 1$

Example 6

How many three letter words can be formed from the sample space {a, b, c, d, e, f}

Solution

Total number of letters = 6 and r = 3

Total number of worms = $^6P_3$

$= \frac{6!}{(6-3)!} = \frac{6!}{3!} = \frac{6\ x\ 5\ x\ 4\ x\ 3!}{3!} = 120$ways

Example 7

Find the possible number of ways of arranging 3 letters from the word MANGOES

Solution

Total number of letter in the word = 7

and r = 3

Number of ways $^7P^3 = \frac{7!}{(7-3)!}$

$= \frac{7\ x\ 6\ x\ 5\ x\ 4\ x\ 3!}{3!} = 840$ways

Example 8

Find number of ways of arranging six boys from a group of 13

Solution

Number of arrangements = ${}^{13}P_6$

$=\frac{13!}{(13-6)!} = \frac{13!}{7!}$

$= \frac{13x12x11x10\ x9\ x\ 8\ x\ 7!}{7!}$

= 1235520 ways

The number of permutations of n objects of which r are alike

The number of permutations of n objects of which r are alike is given by $\frac{n!}{r!}$

Example 9

Find the number of arranging in a line the letters B, C, C, C, C, C, C

The number of ways of arranging the seven letters of which of which 6 are alike $= \frac{7!}{6!} = \frac{7\ x\ 6!}{6!} = 7$ ways

The number of ways of permutations on n objects of which p of one type are alike, q of the second type are alike, r of the third type are alike, and so on.

The number of ways of permutations on n objects of which p of one type are alike, q of the second type are alike, r of the third type are alike given by $\frac{n!}{p!\ x\ q!\ x\ r!}$

Example 10

Find the possible number of ways of arranging the letter of the word MATHEMATICS in line

Solution

The word MATHEMATICS has 11 letters and contains 2 M, 2A and 2T repeated

The number of ways = $\frac{11!}{2!x\ 2!\ x\ 2!}$

= $\frac{11\ x\ 10\ x\ 9\ \ x\ 8\ x\ 7\ x\ 6\ x\ 5\ x\ 4x3\ x\ 2\ x\ 1}{2\ x\ 2\ x\ 2}$

= 4,989,600

Example 11

Find the possible number of ways of arranging the letter of the word 'MISSISSIPPI' in line

Solution

The word 'MISSISSIPPI' has 10 letters with 4I, 4S, and 2P

The number of ways = $\frac{11!}{4!x\ 4!\ x\ 2!} = 34650$

The number of permutations of the like and unlike objects with restrictions

One should be cautious when handling these problems

Example 12

Find the possible number of ways of arranging

The letters of the word MINIMUM if the arrangement begins with MMM?

Solution

There is only one way of arranging MMM

The remaining contain four letters with 2I can be arranged in

$\frac{1\ x\ 4!}{2!} = \frac{1\ x\ 4\ x\ 3\ x\ 2!}{2!} = 12$ ways

Example 13

(a) How many 4 digit number greater than 6000 can be formed from 4, 5, 6, 7, 8 and 9 if:

(i) Repetitions are allowed

Solution

The first digit can be chosen from 6, 7, 8 and 9, hence 4 possible ways, the 2nd, 3rd and 4th are chosen from any of the six digits since repetitions are allowed

position	1st	2nd	3rd	4th
selections	4	6	6	6

Number of ways

= 4 x 6 x 6 x 6 = 864 ways

(ii) Repetition are not allowed

The first can be chosen from 6, 7, 8 and 9, hence 4 possible ways, the 2nd from 5, 3rd from 4 and 4th from 3 since no repetitions are allowed

position	1st	2nd	3rd	4th
selections	4	5	4	3

Number of ways
= 4 x 5 x 4 x 3 = 240 ways

(b) Find how many four digit numbers can be formed from the six digits 2, 3, 5, 7, 8 and 9 without repeating any digit.
Find also how many of these numbers
(i) Are less than 7000
(ii) Are odd

Solution

position	1^{st}	2^{nd}	3^{rd}	4^{th}
selections	6	5	4	3

Total number of ways = 6 x 5 x 4 x 3 = 360

(i) The 1^{st} number is selected from three (2, 3, 5), the 2^{nd} number from 5, the 3^{rd} from 4 and the 4^{th} from 3 digits

position	1^{st}	2^{nd}	3^{rd}	4^{th}
selections	6	5	4	3

Total number of less than 7000

= 3 x 5 x 4 x 3 = 180

(ii) The last number is selected from four odd digits (3, 5, 7, and 9), the 1^{st} number selected from five remaining, 2^{nd} from 4 and 3^{rd} from 3

position	1^{st}	2^{nd}	3^{rd}	4^{th}
selections	5	4	3	4

Total number of odd numbers formed

= 5 x 4 x 3 x 4 = 240

(c) How many different 6 digit number greater than 500000 can be formed by using the digits 1, 5, 7, 7, 7, 8
Solution
The 1^{st} digit is selected from five (5, 7, 7, 7, 8), the 2^{nd} from remaining five, 3^{rd} from four, 4^{th} from three, 5^{th} from two and 6^{th} from one

1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6th
5	5	4	3	2	1

Total number = $\frac{5 \: x \: 5 \: x \: 4 \: x \: 3 \: x \: 2 \: x \: 1}{3!}$ = 100
NB. The number is divided by 3! Because 7 appears three times

(d) How many odd numbers greater than 60000 can be formed from 0, 5, 6, 7, 8, 9, if no number contains any digit more than once
Solution
Considering six digits
Taking the first digit to be odd, the first digit is selected from 3 digits (5, 7, 9) and the last is selected from 2 digits

1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6th
3	4	3	2	1	2

Number of ways = 3 x 4 x 3 x 2 x 1 x 2
= 144
Taking the first digit to be even, the first digit is selected from 2 digits (6, 8) since the number should be greater than 60000 and the last is selected from 2 odd digits

1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6th
2	4	3	2	1	3

Number of ways = 3 x 4 x 3 x 2 x 1 x 2
= 144
Considering five digits
Taking the first digit to be odd, the digit greater than 6 are 7 and 9 so first digit is selected from 2 digits and the last is selected from 2 digits

1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
2	4	3	2	2

Number of ways = 2 x 4 x 3 x 2 x 2
= 96
Taking the first digit to be even, the first digit is selected from 2 digits (6, 8) since the number should be greater than 60000 and the last is selected from 3 odd digits (5, 7, 9)

1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
2	4	3	2	3

Number of ways = 2 x 4 x 3 x 2 x 3
= 144
The total number of selections
= 144 + 144 + 96 + 144 = 528

Example 14

The six letter of the word LONDON are each written on a card and the six cards are shuffled and placed in a line. Find the number of possible arrangements if

(a) The middle two cards both have the letter N on them
Solution
If the middle letter are NN, the we need to find the number of different arrangements of the letter LODO. With the 2O's, the number of arrangements = $\frac{4!}{2!} = 12$

(b) The two cards with letter O are not adjacent and the two cards with letter N are also not adjacent
Solution
If the two cards are not adjacent, the number of arrangements = Total number of arrangements of the word LONDON – number of arrangements when the two letters are adjacent
= = $\frac{6!}{2!2!} - 24 = 156$

Example 15

In how many different ways can letters of the word MISCHIEVERS be arranged if the S's cannot be together

Solution

There are 11 letters in the word MISCHIEVERS with 2S's, 2I's and 2E's

Total number of arrangements

= $\frac{11!}{2!2!2!}$ = 4989600

If S's are together, we consider them as one, so the number of arrangements

= $\frac{10!}{2!2!}$ = 907200

∴ the number of possible arrangements of the word MISCHIEVERS when S's are not together

= 4989600 – 907200 = 4082400

The number of permutation of n different objects taken r at a time, if repetition are permitted

Example 16

How many four digit numbers can be formed from the sample space {1, 2, 3, 4, 5} if repetitions are permissible

Solution

The 1st position has five possibilities, the 2nd five, the 3rd five, the 4th five

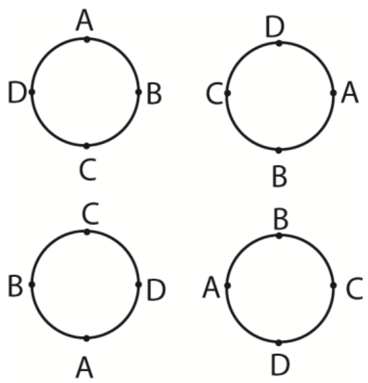
Number of permutations = 5 x 5 x 5 x 5 = 625

Circular permutations

Here objects are arrange in a circle

The number of ways of arranging n unlike objects in a ring when clockwise and anticlockwise are different.

Consider four people A, B, C and D seated at a round table. The possible arrangements are as shown below



With circular arrangements of this type, it is the relative positions of the objects being arranged which is important. The arrangements of the people above is the same. However, if the people were seated in a line the arrangements would not be the same, i.e. A, B, C, D is not the same as D, A, B, C. When finding the number of different arrangements, we fix one person say A and find the number of ways of arranging B, C and D.

Therefore, the number of different arrangements of four people around the table is 3!

Hence the number of different arrangements of n people seated around a table is (n – 1)!

Example 17

(a) Seven people are to be seated around a table, in how many ways can this be done
Solution
The number of ways = (7 – 1)! = 6!
= 720

(b) In how many ways can five people A, B, C, D and E be seated at a round table if
(i) A must be seated next to B

Solution

If A and B are seated together, they are taken as bound together. So four people are considered

The number of ways = (4 – 1)! = 3! = 6

The number of ways in which A and B can be arranged = 2

The total number of arrangements

= 6 x 2 = 12 ways

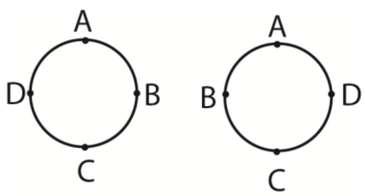
(ii) A must not seat next to B

If A and B are not seated together, then the number of arrangements = total number of arrangements – number of arrangements when A and B are seated together

= (5 – 1)! – 12 = 12 ways

The number of ways of arranging n unlike objects in a ring when clockwise and anticlockwise arrangements are the same

Consider the four people above, if the arrangement is as shown below



Then the above arrangements are the same since one is the other viewed from the opposite side

The number of arrangements = $\frac{3!}{2}$ = 3ways

Hence the number of ways of arranging n unlike objects in a ring when clockwise and anticlockwise arrangements are the same = $\frac{(n-1)!}{2}$

Example 18

A white, a blue, a red and two yellow cards at arranged on a circle. Find the number of arrangements if red and white cards are next to each other.

Solution

If red and white cards are next to each other, they are considered as bound together. So we have four cards. Since anticlockwise and clockwise arrangements are the same and there are two yellow cards, the number of arrangements = $\frac{(4-1)!}{2\ x\ 2!} = \frac{3!}{2\ x\ 2!}$

The number of ways of arranging red and white cards = 2

Total number of ways of arrangements

=$\frac{3!}{2\ x\ 2!}\ x\ 2$ = 3

Revision exercise 1

1. In how many ways can the letters of the words below be arranged
   (a) Bbosa (5!]
   (b) Precious [8!]
2. How many different arrangements of the letters of the word PARALLELOGRAM can be made with A’s separate [83160000]
3. How many different arrangements of the letters of the word CONTACT can be made with vowels separated? [900]
4. How many odd numbers greater than 6000 can be formed using digits 2, 3, 4, 5 and 6 if each digit is used only once in each number [12]

5. Three boys and five girls are to be seated on a bench such that the eldest girl and eldest boy sit next to each other. In how many ways can this be done [2 x 7!]
6. A round table conference is to be held between delegates of 12 countries. In how many ways can they be seated if two particular delegates wish to sit together [2 x 10!]
7. In how many ways can 4 boys and 4 girls be seated at a circular table such that no two boys are adjacent [144]
8. How many words beginning or ending with a consonant can be formed by using the letters of the word EQUATION? [4320]

Combinations

A combination is a selection of items from a group not basing on the order in which the items are selected

Consider the letters A, B, C, D

The possible arrangements of two letters chosen from the above letters are

AB, AC, AD, BA, BC, BD, CA, CB, CD, DA, DB, DC. AS seen earlier, the total number of arrangements of the above letters is expressed as $\frac{4!}{(4-2)!} = \frac{4!}{2!} = 12$

However, when considering combinations, the grouping such as AB and BA are said to be the same groupings such as CA and AC, AD and DA, etc.

So the possible combinations are AB, AC, AD, BC, BD, CD which is six ways.

Thus the number of possible combinations of n items taken r at a time is expressed as ^nC_r or $\binom{n}{r}$ which is defined as $^nC_r = \frac{n!}{(n-r)!r!}$ where $r \leq n$

Hence the number of combinations of the above letters taken two at a time is

$$\binom{4}{2} = \frac{4!}{2!2!} = 6$$

Example 19

A committee of four people is chosen at random from a set of seven men and three women

How many different groups can be chosen if there is at least one

(i) Woman on the committee

Solution

Possible combinations

7 men	3 women
3 2 1	1 2 3

The number of ways of choosing at least one woman

$$=\binom{7}{3} x \binom{3}{1} + \binom{7}{2} x \binom{3}{2} + \binom{7}{1} x \binom{3}{3}$$

$$=\frac{7!}{3!4!} x \frac{3!}{1!2!} + \frac{7!}{2!5!} x \frac{3!}{2!1!} + \frac{7!}{1!6!} x \frac{3!}{3!0!} = 175$$

(ii) Man on the committee

7 men	3 women
1 2 3 4	3 2 1 0

The number of ways of choosing at least one man

$$\binom{7}{1} x \binom{3}{3} + \binom{7}{2} x \binom{3}{2} + \binom{7}{3} x \binom{3}{1} + \binom{7}{4} x \binom{3}{0}$$

$$=\frac{7!}{1!3!} x \frac{3!}{0!3!} + \frac{7!}{5!2!} x \frac{3!}{1!2!} + \frac{7!}{4!3!} x \frac{3!}{2!1!} + \frac{7!}{3!4!} x \frac{3!}{3!0!}$$

=210

Example 20

A group of nine has to be selected from ten men and eight women. It can consist of either five men and four women or four men and five women. How many different groups can be chosen?

Solution

Possible combination

10 men	8 women
5 4	4 5

Number of groups = $\binom{10}{5} x \binom{8}{4} + \binom{10}{4} x \binom{8}{5}$

= $\frac{10!}{5!5!}$ x $\frac{8!}{4!4!} + \frac{10!}{6!4!}$ x $\frac{8!}{5!3!}$ = 29400

Example 21

A team of six is to be formed from 13 boys and 7 girls. In how many ways can the team be selected if it must consist of

(a) 4 boy and 2 girls

13 boys	7 girls
4	2

$\binom{13}{4}.\binom{7}{2} = \frac{13!}{9!4!} x \frac{7!}{5!2!} =$ 15015

(b) At least one member of each sex

Possible combinations

13 boys	7 girls
5	1
4	2
3	3
2	4
1	5

$= \binom{13}{5}.\binom{7}{1} + \binom{13}{4}.\binom{7}{2} + \binom{13}{3}.\binom{7}{3} + \binom{13}{2}.\binom{7}{4} + \binom{13}{1}.\binom{7}{5}$

$= \frac{13!}{8!5!}.\frac{7!}{6!1!} + \frac{13!}{9!4!}.\frac{7!}{5!2!} + \frac{13!}{10!3!}.\frac{7!}{4!3!} + \frac{13!}{11!2!}.\frac{7!}{3!4!} + \frac{13!}{12!1!}.\frac{7!}{2!5!}$

= 37037

Example 22

A team of 11 players is to be chosen from a group of 15 players. Two of the 11 are to be randomly elected a captain and vice-captain respectively. In how many ways can this be done?

Number of ways of choosing 11 players from 15 = $\binom{15}{11}$

A captain will be elected from 11 players and a vice-captain from 10 players

Total number of selection = $\binom{15}{11} x\ 11\ x\ 10$

$= \frac{15!}{4!11!}$ x 11 x 10 = 150150

Example 23

(a) Find the number of different selections of 4 letters that can be made from the word UNDERMATCH.
Solution
There are 10 letters which are all different
Number of selections of 4 letters from 10 is given by $\binom{10}{4} = \frac{10!}{(10-4)!4!} = \frac{10!}{6!4!} = 210$

(b) How many selections do not contain a vowel?
Solution
Number of vowels in the word = 2
Number of letters not vowels = 8
Number of selections of 4 letters from 10 without containing a vowel = selecting 4 letters from 8 consonants =
$\binom{8}{4} = \frac{8!}{(8-4)!4!} = \frac{8!}{4!4!} = 70$

Example 24

In how many ways can three letters be selected at random from the word BIOLOGY is selection

(a) Does not contain the letter O
Solution
Number of selections without the letter O = number of ways of choosing three letters from B, I, L, G, y
$= \binom{5}{3} = \frac{5!}{2!3!} = 10$

(b) Contain only the letter O

Solution

Number of selections with one letter O = number of ways of choosing two letters from B, I, L, G, y

$= \binom{5}{2} = \frac{5!}{3!2!} = 10$

(c) Contains both of the letters O

Solution

Number of selections with two letter O = number of ways of choosing one letter from B, I, L, G, y

$= \binom{5}{1} = \frac{5!}{4!1!} = 5$

Example 25

In how many ways can four letters be selected at random from the word BREAKDOWN if the letters contain at least one vowel?

Solution

Vowels: E, A, O (3)

Consonants: B, R, K, D, W, N (6)

Consonants (6)	Vowels (3)
3	1
2	2
1	3

Number of selection of four letters with at least one vowel

$= \binom{6}{3}.\binom{3}{1} + \binom{6}{2}.\binom{3}{2} + \binom{6}{1}.\binom{3}{3} = 111$

Example 26

How many different selections can be made from the six digits 1, 2, 3, 4, 5, 6

Solution

Note: this an open questions because selections can consist of only one digit, two digits, three digits, four digits, five digits or six digits

Number of selection of 1 digit = $^6C_1$ = 6

Number of selection of 2 digits = $^6C_2$ = 15

Number of selection of 3 digits = $^6C_3$ = 20

Number of selection of 4 digits = $^6C_4$ = 15

Number of selection of 5 digits = $^6C_5$ = 6

Number of selection of 6 digits = $^6C_1$ = 1

Total number of selections

= 6 + 15 + 20 + 15 +6 + 1 = 63

This approach is tedious for a large group of objects.

The general formula for selection from n unlike objects is given by $2^n - 1$.

For the above problems, number of selections = $2^6 - 1$ = 63

Example 26

How many different selections can be made from 26 different letters of the alphabet?

Number of selection = $2^{26} - 1$

= 67,108,863

Cases involving repetitions

Suppose we need to find the number of possible selections of letters from a word containing repeated letters, we take the selections mutually exclusive

Example 27

How many different selections can be made from the letters of the word CANADIAN?

Solution

There are 3A's, 2N's and 3 other letters

The A's can be dealt with in 4 ways (either no A, 1A's, 2A's or 3A's)

The N's can be dealt in 3 ways (no N, 1N, or 2N's)

The C can be dealt with in 2 ways (no C, 1C)

The D can be dealt with in 2 ways (no D, 1D)

The I can be dealt with in 2 ways (no I, 1I)

The number of selections

= 4 x 3 x 2 x 2 x 2 – 1 = 95

Example 28

How many different selections can be made from the letters of the word POSSESS?

Solution

There are 4S's and 3 other letters

The S's can be dealt in 5 ways (no S, 1S, 2N's, 3S's, 4S's, or 5S's)

The P can be dealt with in 2 ways (no P, 1P)

The O can be dealt with in 2 ways (no O, 1O)

The E can be dealt with in 2 ways (no E, 1E)

Total number of selections = 5 x 2 x 2 x 2 – 1

= 39

Cases involving division into groups

The number of ways of dividing n unlike objects into say two groups of p and q where p + q = n is given by $\frac{n!}{p!q!}$

For three groups of p, q and r provided p + q + r = n

Number of ways of division = $\frac{n!}{p!q!r!}$

However, for the two groups above, if p = q then the number of ways of division = $\frac{n!}{p!p!2!}$

For three groups where p = q = r

then the number of ways of division = $\frac{n!}{p!p!p!3!}$

Example 29

The following letters a, b, c, d, e, f, g, h, I, j, k, l are to be divided into groups containing

(a) 3, 4, 5
(b) 5, 7
(c) 6, 6
(d) 4, 4, 4 letters. In how many ways can this be done?

Solution

(a) Number of ways = $\frac{12!}{3!4!5!}$ = 27720
(b) Number of ways = $\frac{12!}{5!7!}$ = 792
(c) Number of ways = $\frac{12!}{6!6!2!}$ = 462
(d) Number of ways = $\frac{12!}{4!4!4!3!}$ =5775

Example 30

Find the number of ways that 18 objects can be arranged into groups if there are to be

(a) Two groups of 9 objects each
(b) Three groups of 6 objects each
(c) 6 groups of 3 objects each
(d) Three groups of 5, 6 and 7 objects each

Solution

(a) Number of ways = $\frac{18!}{9!9!2!}$ = 24310
(b) Number of ways = $\frac{18!}{6!6!6!3!}$ = 2858856
(c) Number of ways = $\frac{18!}{3!3!3!3!3!3!6!}$ =190590400
(d) Number of ways = $\frac{18!}{5!6!7!}$ = 14702688

Example 31

(a) Find how many words can be formed using all letters in the word MINIMUM.

Solution

Number of ways of arranging the letters = 7!

There are 3M's and 2I's

Number of words formed = $\frac{7!}{3!2!} = 420$

(b) Compute the sum of four-digit numbers formed with the four digits 2, 5, 3, 8 if each digit is used only once in each arrangement

Solution

Number of ways of arranging a four digit number = 4!

Sum of any four digit number formed = 2 + 5 + 3 + 8 = 18

Total sum of four digit numbers formed

= 18 x 4! = 432

(c) A committee consisting of 2 men and 3 women is to be formed from a group of 5 men and 7 women. Find the number of different committees that can be formed. If two of the women refuse to serve on the same committee, how many committees can be formed?

Solution

The committees formed = $^5C_2.^7C_3$

= 10 x 35 = 350

Suppose two women are to serve together, we take them as glued together, so the number of committees =$^5C_2.^6C_3$ = 200

Number of committees in which two women refuse to serve together = 350 – 200 = 150

Revision exercise 2

1. (a) Find the number of different selection of 3 letters that can be made from the word PHOTOGRAPH. [53]
   (b) How many of these selections contain no vowel [18]
   (c) How many of these selections contain at least one vowel? [35]
2. (a) find the number of different selections of 3 letters that can be made from the letters of the word SUCCESSFUL.[36]
   (c) How many of these selections contain only consonants [11]
   (d) How many of these selections contain at least one vowel [25]
3. (a) Find the value of n if ${}^{n}P_4 = 30{}^{n}C_5$ [8]
   (b) How many arrangement can be made from the letters of the name MISSISSIPPI
      (i) when all the letters are taken at a time [34650]
      (ii) If the two letters PP begin every word [630 ways]
   (c) Find the number of ways in which a one can chose one or more of the four girls to join a discussion group [15 ways]
4. Find in how many ways 11 people can be divided into three groups containing 3, 4, 4 people each. [5775]
5. A group of 5 boys and 8 girls. In how many ways can a team of four be chosen, if the team contains
   (a) No girl [5]
   (b) No more than one girl [85]
   (c) At least two boys [365]
6. Calculate the number of 7 – letter arrangements which can be made with the letters of the word MAXIMUM. In how many of these do all the 4 consonants appear next to each other? [840, 96]
7. In how many ways can a club of 5 be selected from 7 boys and 3 girls if it must contain
   (a) 3 boys and 3 girls [105]
   (b) 2 men and 3 girls [21]
   (c) At least one girl [231]
8. How many different 6 digit numbers greater than 400,000 can be formed form the following digits 1, 4, 6, 6, 6 7? [100]

Thank you

Dr. Bbosa Science

KIIRA COLLEGE BUTIKI

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

LOCK DOWN REVISION QUESTIONS

SECTION A (40 marks)

1. Solve the inequality

   $\frac{x(x+2)}{x-3} \leq x+1$ (5 marks)

2. Show that the line $\frac{x-2}{2} = \frac{y-2}{-1} = \frac{z-3}{3}$

   Is parallel to the plane 4x-y-3z=4 and find the perpendicular distance of the line from the plane. (5 marks)

3. Solve the equation

   2tan x-3cot x=1

   For $0^o \leq x \leq 360^o$ (5 marks)

4. Calculate the co-ordinates of the point of the intersection of the curve

   $\frac{x}{y} + \frac{6y}{x} = 5$ and $2y = x - 2$ (5 marks)

5. The tangent to the curve $y = 2x^2 + ax + b$ at the point (-2,11) is perpendicular to the line $2y = x + 7$. Find the value of a and b. (5 marks)

6. Evaluate $\int_{o}^{\frac{\pi}{3}} cos3xcos2xdx$ (5 marks)

7. Given that φ in a root of the equation $x^2 - 2x + 3 = 0$ show that $\varphi^3 = x - 6$

   (5 marks)

8. A spherical balloon is being inflated by gas being pumped at the constant rate of $200cm^3$ per second. What is the rate of increase of the surface area of the ballon when its radius is 100cm? (5 marks)

SECTION B (60 MARKS)

9. (a) $If\ (x+1)^2\ is\ factor\ of\ 2x^4+7x^3+6x^2+Ax+b,$ find the value of A and B. (5 marks)

(b) Prove that, if the equations $x^2+ax+b=0\ and\ cx^2+2ax-3b=0$ have a common root and neither a and b is zero, then

$b=\frac{5a^2(c-2)}{(c+3)^2}$ (7 marks)

10. (a) Given that $y=loge(\frac{3+4cosx}{4+cosx})$ find $\frac{dy}{dx}$ in the simplest form. (7 marks)

(b) $If\ y=e^{4x}cos3x, prove\ that\ \frac{d2y}{dx^2}-8\frac{dy}{dx}+25y=o$ (7 marks)

11. (a) Given that $z=cos\theta_{sin\theta}, where\ \theta\neq\pi, show\ that\ \frac{2}{1+z}=1-itan\frac{1}{2}\theta.$

(6 marks)

(b) The polynomial $p(z)=z^4-3z^3+7z^2+21z-26\ has\ 2+3i$

$as\ one\ of\ the\ roots. Find\ the\ other\ three\ roots\ of\ the\ equation\ p(z)=o$

(6 marks)

12. (a) A right circular cone with semi vertical angle θ is inscribed in a sphere of radius γ, with its vertex and rim of its base on the surface of the sphere. Prove that its volume is $\frac{8}{3}\pi r^3cos^4\theta sin^2\theta.$ (6 marks)

(b) If r in constant and θvaries, show that the limits within which this volume must lie is $0<v<\frac{32\pi r^3}{81}1$ (6 marks)

13. (a) In any triangle ABC, prove that $tan\frac{1}{2}(B-C)=\left(\frac{b-c}{b+c}\right)tan\frac{1}{2}(B+C)$

(6 marks)

(b) In a particular triangle the angle A is 51^o and b=3c. Find the angle B to the nearest degree. The area of this triangle in $0.47m^2$. Find side a to three decimal places.

14. (a) The points A and B have position vector i-2jtk and 2ijk respectively. Given that $0c=\boldsymbol{\lambda}$OA $+\mu OB\ and\ OC\ is\ perpendicular\ to\ OA, find\ the$ Ratio of $\boldsymbol{\lambda}$ to μ.

Write down the vector equation of the line, L through A which is perpendicular to OA. Find the position vector of P, the point of intersection of Land OB. (12 marks)

15. (a) Determine the equation of the normal to the eclipse $x\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at a point p(a cos θ, bsin θ. (6 marks)

(b) If the normal at p meets the x – axis at A and the y – axis at B, Find the locus of the midpoint of AB. (6 marks)

16. (a) Solve the differential equation $x\frac{dy}{dx}=y+x^2(cosx+sinx),$ given that $y=o\ when\ x\frac{\pi}{2}$ (5 marks)

(b) The rate of decay of a radioactive substance is proportional to the amount A remaining at any time t. If initially the amount was Ao and if the time taken for the amount of substance to become ½ Ao is T, find A at that time.

Find the time taken for the amount remaining to be reduced to $\frac{1}{20}Ao$

(7 marks)

KIIRA COLLEGE BUTIKI

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

LOCK DOWN REVISION QUESTIONS 2020

SECTION A (40 MARKS)

1. Solve the simultaneous equations;

   $x + y = 4$

   $x^2 + y^2 - 3xy = 76$ *(05 marks)*

2. Solve the equation; $\sqrt{3}\ \sin\theta - \cos\theta + 2 = 0$ for $0 < 0 < 2\pi.$ *(05 marks)*

3. Find the equations of the lines which pass through the point A(3, -2) and makes an angle θ with the line $2x - 3y – 4 = 0$, where $\tan\theta = 2$. *(06 marks)*

4. Show that $\frac{(\sqrt{3} - i)^5}{\sqrt{3} + i} = -16$ *(05 marks)*

5. If $y = A\,x^k$, where A and K are non – zero constants , find the values of K such that; $x^2 \frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = 0$ *(05 marks)*

6. Using the substitution $x = e^t$, evaluate the $\int_1^e \frac{3 - 1nx}{x^2}\ dx.$ *(05 marks)*

7. Given that A and B are points whose position vectors are **a** = 2**i** + **k** and **b** = **i** - **j** + 3**k** respectively.
   Determine the position vector of the point that divides AB in the ratio -4 : 1 *(04 marks)*

8. Find the area bounded by the three curves $y = x^2$, $y = \frac{1}{4}x^2$ and $y = \frac{1}{x^2}$ in the first quadrant. *(05 marks)*

SECTION B (60 MARKS)

9. (a) Find $\int \frac{1}{x^3\ \sqrt{x^2 - 4}}\, dx$ *(06 marks)*

(b) Evaluate $\int_3^4 \frac{x^3}{x^2 - x - 2}\, dx$ *(06 marks)*

10. (a) The eighth term of an arithmetic progression is twice the fourth term, and the sum of the eight terms is 30. Find the

(i) first four terms, *(06 marks)*

(ii) sum of the first 12 terms, of the progression *(02 marks)*

(b) Find the number of ways in which the letters of the word STATISTICS can be arranged in a straight line so that,

(i) the last two letters are both Ts. *(02 marks)*

(ii) all the three **Ss** must be together *(02 marks)*

11. (i) Given that the roots of the equation $ax^2 + bx + c = 0$ are α and β. Show that $a^2 = b^2 - 4ac$ if $\alpha - \beta = 1$. *(06 marks)*

(ii) Find a quadratic equation whose roots are $(\alpha + \alpha\beta)$ and $(\beta + \beta\alpha)$ in terms of a, b and c. *(06 marks)*

12. (a) Differentiate with respect to x,

(i) $2^{COS\ X^2}$ *(03 marks)*

(ii) $\log_e \left(\frac{(1+x)\, e^{-2x}}{1 - x}\right)^{½}$ *(03 marks)*

(b) (i) Determine the equation of the normal to the curve $y=\frac{1}{x}$ at the point x = 2. *(03 marks)*

(ii) Find the coordinates if the other point where the normal meets the curve again

(03 marks)

13. (a) Given the points A (3, 1, 2) and B (2, -2, 4), find the sin e of the angle BOC.
Hence determine the area of triangle AOB. Where O is the origin. *(06 marks)*

(b) Show that the line $\frac{x-2}{2} = \frac{2-y}{1} = \frac{3-z}{-3}$ is parallel to the plane r .(4**i** – **j** – 3**k**) = 4.
Hence find the perpendicular distance between the line and the plane.
(06 marks)

14. (a) Show that for any triangle ABC,
$\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ *(05 marks)*

(b) Prove that $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$, hence solve the equation

$\tan(x - 45^0) = 6\tan x$, where $-180^0 \le x \le 180^0$ *(07 marks)*

15. (a) Find the equation and radius of a circle passing through the points A (0,1), B (0, 4) and C (2 , 5). *(05 marks)*

(b) A circle passes through the point P(1, -4) and is tangent to the y-axis. If its radius is 5 units, find its equation *(07 marks)*

16. (a) Given that y = 0 when x = 0, solve the equation $\frac{dy}{dx} = 2y + 3$, expressing y as a function of x. *(05 marks)*

(b) When a uniform rod is heated it expands in such a way that the rate of increase of its length, l, with respect to the temperature, θ^0 C, is proportional to the length. When the temperature is 0^0C the length of the rod is L. Given that the length of the rod has increased by 1% when the temperature is 20^0C, find the value of θ at which the length of the rod has increased by 5%. *(07 marks)*

WAKISSHA JOINT MOCK EXAMINATIONS 2015
UGANDA ADVANCED CERTIFICATE OF EDUCATION
MARKING GUIDE
P425/1
MATHEMATICS
PAPER 1
JULY/AUGUST 2015

1. $X=5+\frac{\sqrt{3}}{2}\cos\Theta$ .... (1)		
$Y=-3\frac{\sqrt{3}}{2}\sin\Theta$ .... (2)		
From 1 $X-5=\frac{\sqrt{3}}{2}\cos\Theta$		
$\left(\frac{2X-10}{\sqrt{3}}\right)=\cos\Theta$		
$\sqrt{3}$		
$\cos^2\Theta=\left(\frac{2X-10}{\sqrt{3}}\right)^2$		
From (2) $Y+3=\frac{\sqrt{3}}{2}\sin\Theta$		
$\sin\Theta=\frac{2(Y+3)}{\sqrt{3}}$	B1	B1 – for $Co^2\Theta$ and $\sin\Theta$
$\sin^2\Theta=\frac{(2Y-6)^2}{3}$		Or
	M1	$(x-5)^2=\frac{3}{4}\cos^2\Theta$.........(1) B1
$\cos^2\Theta+\sin^2\Theta=2\frac{(2X-10)^2}{3}+\frac{(2Y-6)^2}{3}$		$(y+3)2=\frac{3}{4}\sin^2\Theta$.........(2)
$(2X-10)^2+(2Y-6)^2=3$		(1) + (2)
$4X^2-40X+100+4Y2-24Y+36=3$		$(x-5)^3+(y+3)^2=\frac{3}{4}$ M1
$4X^2+4Y^2-40X-24Y+133=0$		$4x2+4y2-40x-24y+130=0$ A1
$X^2+Y^2-10X+6Y-133-0$		Is locus of a circle B1
$(X-5)^2+(Y+3)^2=\frac{3}{4}$		The radius $=\frac{\sqrt{3}}{2}$ and centre (5, -3) B1
Locus is a circle with centre	B1	
(5 – 3) and radius $\sqrt{3}$	B1	
	05	

2. 3^{2n+2}-8n-9. Let au= 3^{2n+2}-8n-9. For n=1 $a_1=3^4$-8-9=64 thus divisible. For n=k. ak=3^{2X+2}-8k-9.$\Rightarrow$9k = $3^{2k}.3^2$-8k-9 9k=9($3^{2\pi}$) - 8k-9. 32k = $\frac{9k+8k+9}{9}$ For = k+1 9R+1 = $3^{2(k+1)+2}$-8(k+1)-9 9R+1=$3^{2\pi+4}$-8k –8-9$\Rightarrow$9K+1=81(3^{4k}) -8k - 17 Subst.(3) in (4) 81($\frac{9k+8k+9}{9}$).8k-17 9k+1=9ak+72R+81-8K-17. 9K+1= 9AK+64R+64. 9K+1 = 64($\frac{9ak}{64}$+k+1) Ak+1=64Bfor B=$\frac{9ak}{64}$+C+1 Since its divisible for n= π41,n=c thus this also divisible for all integral values of n.	B1 B1 B1 B1 05	
3. 6Cos2X+777=7Sin2X using t = tan x $6\left(\frac{1-t^2}{1+t^2}\right)+7=7\left(\frac{2t}{1+t^2}\right)$ 6(1-t^2) +7(1+t^2) =14t.$\Rightarrow t^2$-14t+13=0 (t-1) (t-13)=0 t=1 t=13 for t=1 tan X=1 X=45^0,225^0 For t=13 tan x=13 x=85.6^0, 265^- 6^0 Or 7Sin2x-6Cos2X=7 R Sin(2x∝)= 7Sin2X-6CosR R Cos∝=7 and R Sin∝=6 R=$\sqrt{49}+36$ = S85 tan∝=$\frac{6}{7}$ $\sqrt{85}$ Sin(2x-∝)=7 Sin(2x-∝)=$\frac{7}{\sqrt{85}}$ 2x = $\sin^{-1}\left(\frac{7}{85}\right)$+$\tan^{-1}\left(\frac{6}{7}\right)$X = A1A1 = 05	M1 M1 A1 A1 A1	For substitution For method

4. $|X+3a|<2|X-2a|$

$(X+3a)^2<4(x-2a)^2$

$X^2+6ax+9a2<4(X^2-4aX+4a)$

$3x^2-22ax+7a^2>0$

$(3x-a)(x-7a)>0$ — A1

	$X<\frac{1}{3}a$	$\frac{1}{3}a<x<7a$	$X>7a$
3x-a	-	+	+
x-7a	-	-	+
	+	-	+

$\therefore$ The solution is $X<\frac{1}{3}a$ and $x>7a$ — A1 — A1 for each range of solution

---

5. $\int_{\sqrt{3}}^{\infty} \frac{dx}{x\sqrt{1+x^2}}$

Method 1

Let $u=\sqrt{1+x^2}$

x	$\sqrt{3}$	∞
u	2	∞

$du = \frac{1}{2}(1+x2)^{-1/2}.2xdx.$ — B1

$du = \frac{dx}{\sqrt{1+x^2}}dx$

$I = \int_2^{\infty} \frac{1}{U^2-1}\frac{U}{X}\ du$

$I = \int_2^{\infty} \frac{1}{U^2-1}\ du$

Let $\frac{1}{U^2-1} = \frac{A}{u-1} + \frac{B}{u+1}$ — B1

$A=\frac{1}{2}$, $B=\frac{-1}{2}$

$$I = \frac{1}{2}\left[In\left(\frac{u-1}{u+1}\right)\right]_2^{\alpha}$$ — M1

$=\frac{1}{2}(0-in\frac{1}{3})$

$=\frac{1}{2}in3$ — A1

05

Alternative method.

$$\int_{\sqrt{3}}^{\infty} \frac{dx}{x\sqrt{1+x^2}}$$

Let x=tan u — B1

$dx= Sec^2udu$ — (for only changing variablesi.e x to U because it can be worked without involving limits until toward end then we bring limits.)

$I=\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{\tan u\ secu}$ — M1

$=\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{Cos\ U}{Sin\ U}\frac{1}{CosU}\ du.$

$=\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \text{Cosec u du}$

$\left[in(Tan\frac{u}{2})\right]$ $=(0-in\frac{1}{\sqrt{3}}$ $=\frac{1}{2}\ln 3$	B1 A1	
6. (i-λi+4k).(2i-4j+6k)=0 4λ =-26 $\lambda = \frac{-13}{2}$ or -6.5 Using ratio theorem. $OP=(\frac{-3}{-3+2})a + (\frac{-3}{-3+2})b$ 2(i-2j+4k)-2(3i-4j+6k) =3i-6j+12k-6i+8j-12k = -3i+2j	M1 M1 A1 **05**	
7. Let ar, ar^2, ar^3, be the age of the children in order and that of Pondo respectively. ar, ar^2, ar^3=140 $a(i+r+r^2+r^3)=140$ ………..i and a+ar =14 a(i+r) = 14………………..ii ar^2, ar^3=126 ⇒ $ar^2(1+r)$=126……….iii (i)/(ii)gives $r^2=9$ r=+-3 r=3 a(1+3)=14 $a=\frac{7}{2}$ is a root (r+1)(r+3)(r-3)=0 ⇒ r=3, r=-1, and r=-3 are roots. r=3 subset for r=3 in equation (ii) 4a=14 $a=\frac{7}{2}$ Pando's age = $ar^3=\frac{7}{2}x3^3$ =$27x\frac{7}{2}$ giving 94.5years.		

8. YCos2X $\frac{dy}{dx}$= tan x+2, at y=0, X=$\frac{\pi}{4}$ y$\frac{dy}{dx}$= <u>tan x+2</u> Cos^{2}x y$\frac{dy}{dx}$=(tanx+2)sec^{2}x ydy = $\int \tan x\ sec^2xdx +$ $2sec2xdx$ $\frac{y^2}{2}$ =$\frac{1}{2}$tan^{2}x+ tan x+c At y=0, x=$\frac{\pi}{4}$ 0= $\frac{1}{2}$(tan$\frac{\pi}{4}$)^2+2tan$\frac{\pi}{4}$+c = $\frac{1}{2}$(1)^2+2(1)+c C=$\frac{-5}{2}$ $\frac{y^2}{2}$=$\frac{1}{2}$tan^{2}x+ tan x-$\frac{-5}{2}$		
9.		

10. (a)(i) Let Z=x+iy $\lvert Z-i \rvert<3$ $\lvert x+(y-1)i \rvert<3$ $X^2+(y-1)^2<3^2$ This is a circle with centre (0,1) and radius r=3 (ii)This $\pi \leq \arg(z,2) \leq \pi$ This is a region of half lines from (2,0) between $\frac{1}{3}\pi$	B1B1 B1	B1 for stating centre (0,1) B1 for stating $r \leq 3$ For correct sketch and shading
(b) Z =X+IY $\text{Re}\,\frac{Z+i)}{Z+2} = 0$ $\text{Re}\,\frac{x+(y+1)i}{(x+2)+yi}$ $\text{Re}\,\frac{x+(y+1)i\ (x+2)-yi}{(x+2)+yi\ (x+2)-yi}=0$ $\text{Re}\,\frac{x(x+2)-xyi+i(x+2)(y+1)+y(y+1)}{(x+2)^2+y^2}=0$ $\frac{X(x+2)+y(y+1)}{(x+2)^2+y^2}$ $X^2+2x+y^2+y=0$ $(x+1)^2-1+(y+\frac{1}{2})^2-\frac{1}{4}=0$ $(x+1)^2+(y+\frac{1}{2})^2=\frac{5}{4}$ Z=X+iy … on a circle of centre $(-1,\frac{1}{2})$ and radius $R=\frac{\sqrt{5}}{2}$		

11. Sin30=Sin(2Θ+ Θ) = Sin2ΘCosΘ +Cos2Θ Sin Θ (2SinΘCosΘCosΘ)+(1-2Sin^{2}Θ)sinΘ 2SinΘ(1-Sin2Θ)+(1-2Sin^{2}Θ)SinΘ 2SinΘ-2Sin^{3}Θ+SinΘ-2Sin^{3}Θ Sin^{3}Θ = 3SinΘ-4Sin^{3}Θ Let Sin3Θ = $\frac{p}{q}$ ⇒$\frac{p}{q}$=3SinΘ-4 Sin^{-3}Θ 4q Sin^{-3}Θ-3qSinΘ+P=0 If SinΘ= x ⇒4qx^3-3qx+p=8x^3-6x-1=0 Q=2 and P=-1 Sin3Θ $=-\frac{1}{2}$		
3Θ = Sin^{-1}($-\frac{1}{2}$) 210^0,330^0,570^0,690^0,930^0 Q=70^0,110^0,190^0,230^0,310^0 X=Sin700=0.9397. X= Sin1900=-0.1736 X=Sin2300=-0.7660		
12. (a) Y2=4X-8 Can be written as Y2=4(x-2) =4.1(x-2) =4ax X= x-2 a=1 Y^2=4(x-2) is the image of y^2=4x under translation vector $\binom{2}{0}$ New focus = $\binom{1}{0}\binom{2}{0}\binom{3}{0}$ Focus (3,0) New direction = -1+2 X=1		

(b) At P(ap2,2ap) X=ap2 y=2ap $\frac{dx}{dp}$=2aP $\frac{dy}{dp}$=2a. $\frac{dy}{dx}=\frac{2a}{2ap}\frac{1}{p}$ Gradient of tangent at P $=\frac{1}{p}$ Gradient of tangent at Q=$\frac{1}{p}$ Equation of tangent at P $\frac{y-2aP}{x-9p2}=\frac{1}{p}$ x-py+ap2=0……….1 Equation of tangent at Q. x-qy+aq^2=0………..2 solving equations 1 and 2 -py+qy+aq^2-aq^2 =0 (q-p)y+a(p-q)(p-q) =0 Y=a(p+q) Substitute for y in equation 1 X=Pa(P+q)-ap^2 =apq. R is (apq, a(P+q) If R lies on 2x+a=0 Then R satisfies thus equations. 2(apq)+a=0 2pq+1=0 Pq - $\frac{1}{2}$		

Mid-point of PQ is $m\left(\frac{ap^2+aq^2}{2},\frac{2ap+2aq}{2}\right)$. M $\frac{9}{2}$(p^2+q^2), 9(p+q) At M: X=$\frac{9}{2}$(p^2+q^2), y=a(p+q) p^2+q^2 =$\frac{2x}{9}$……………. 3 P+q = $\frac{y}{9}$……………… 4 From 3 $\frac{2x}{9}$= (p+q)2-2pq Pq =- $\frac{1}{2}$ $\frac{2x}{9}$ = (P+q) ^{2}+1…….. 5 Substitution equation 4 in 5 $\frac{2x}{9}$ =($\frac{y}{9}$)^2+1 2ax =y^2+a^2 Y^2=2ax-a^2		

13. (a)

$\sqrt{x} - 3$+$\sqrt{2x+1}$ =$\sqrt{3x+4}$

Squaring both sides.

((x-3)+2$\sqrt{(x-3)(2x+1)}$+(2x+1)=3x+4

⇒$\sqrt{(x-3)(2x+1)}$=3.

2x2-5x-12=0

(x+4)(2x+3)=0

X=4 or x=$\frac{-3}{4}$

Checking.

When x=4 , LHS=RHS

When x=$\frac{-3}{2}$, LHS=RHS

X=4 is only solution.

13(b).

Let the first term of an AP be a and the common difference be d let also the first term of a GP be b and the common ratio be r.

AP= a+(a+d)+(a+2d)+……….+(a+(n-1)d)

G.P= b+br+br^2+br^3+……+ br^{n-1}

⇒Sum of the first terms;

a+b=57………………i

⇒Sum of second terms.

a+d+br = 94

at r=2.

A+d+2b=94………………..ii

Sum of the third term

a+2d+br2=171

at r=2.

a+2d+4b=171………………iii

14. (a) let y= $\frac{2x^2}{x^2+1}$ Dy = $\frac{2(x+h)^2}{\left[(x+h)^2+1\right]}-\frac{2x^2}{x^2+1}$ $\frac{2(x+h)^2}{\left[(x+h)^2+1\right]}-\frac{(x^2+1)-2x^2\left[(x+h)^2\right]+1}{\left[x^2+1\right]}$ $\frac{\left[2\left(x^2 2xh+h^2\right)+1\right]\left[x^2+1\right]-2^2\left[x^2+2xh+h^{2+1}\right]}{\left[(x+h)^2+1\right]\left[x^2+1\right]}$ $\frac{\left[2x^2+4xh+2h^2+1\right]\left[x^2+1\right]-2x^2\left[x^2+2xh+h^2+1\right]}{\left[(x+h)^2+1\right]\left[x^2+1\right]}$ $\frac{2x^4+2x^2+4x^3h+4xh+2x^2h^2+2h^2-2x4-4x^3h-2x^2h^2}{\left[(x+h)^2+1\right]\left[x^2+1\right]}$ $\frac{4xh+2h^2}{\left[(x+h)^2+1\right]\left[x^2+1\right]}$ $\frac{dy}{dx}=\frac{4x+2h}{\left[(x+h)^2+1\right]\left[x^2+1\right]}$		
$\frac{dy}{dx}=\frac{4x}{(x^2+1)(x^2+1)}$ $\frac{dy}{dx}=\frac{4x}{(x^2+1)^2}$		

(b) $X^2+6x+34=(x+3)^2+25$ $25\left[1+\left(\frac{x+3}{5}\right)^2\right]$ Let $\frac{x+3}{5}=\tan\theta$ $\frac{1}{5}=\sec^2\theta\frac{dQ}{DX}$ When x=3 ,tanϴ=0 ϴ=$\tan^{-1}(0)=0$ X=2 tanϴ=$\frac{5}{5}$ ϴ = $\tan^{-1}(1)$ =$\frac{\pi}{4}$ $\int_{-3}^{2}\frac{dx}{x^2+6x+34}=\int_{0}^{\pi/4}\frac{1}{25}\left(\frac{1}{1+\tan^2\theta}\right)5\sec^2\theta dx$ $=\frac{1}{5}\int_{0}^{\pi/4}d\theta$ $=\frac{1}{5}[\theta]_{0}^{\pi/4}$ $=\frac{\pi}{20}$		
15. (a) $\frac{dy}{dx}$ = y+ $\tan\left(\frac{y}{x}\right)$ using y=ux From y=ux $\frac{dy}{dx}$ =u$\frac{d(x)}{dx}+x\frac{d(u)}{dx}$ $\frac{dy}{dx}=u+x\frac{du}{dx}$ $\left(u+x\frac{du}{dx}\right)$ = ux +tan(u) xu+x$^2\frac{dy}{dx}dx=x4+\tan u$		

$x^2\frac{du}{dx}=\tan u$ separating variables $X^2du = \tan dx$ $\frac{du}{\tan u}=\frac{dx}{x^2}$ $\frac{1}{\tan u}du=\frac{dx}{x^2}$ $\frac{\cos u}{\sin u}du=x^{-2dx}$ $\Rightarrow\int\frac{\cos udu}{\sin u}=\int^{x^2dx}$	

In sin u $=\frac{-1}{x}$+c But from y=ux $u = {}^{y}\!/_{x}$ $\Rightarrow$ in sin$\left({}^{y}\!/_{x}\right) = \frac{-1}{x} + c$ Or $In\cos {}^{y}\!/_{x} + \frac{1}{x} + A = 0$	
(b) Let A represent the number of accidents and P represent the number of police deployed. $\frac{dA}{dp} = -kp$. dA=-Kpdp. SdA =-KSpdp. A=-kSpdp A=$\frac{-kp^2}{2} + c$ At A=7,P=2. 7=-k$\frac{4}{2}$+c. 7 =-2k+c……………. 1 At a=1 , P=4 1=-k$\frac{k16}{2} + c$ 1 =-8k+c…………11 K=1, c=9.	

So the equation connecting A and P is A=$\frac{-p^2}{2} + 9$ A= $\frac{-p^2}{2} + 9$ 2A= -p^2+(9X2) P^2 = (2X9)-2A P2=18-2A.	
(i) When there is no policeman P=0. P2 = 18-2A O = 18-2A $\frac{2A}{2} = \frac{18}{2}$ A = 9	

There are 9 accidents if no policeman is deployed.		
(ii) To completely stump out accidents ie A=0 From P2= 18-2A. A=o P2=18 P=√18 = 4.24 =5 5 policemen are required.		
16. $k=\frac{5t}{16+\left(t/a\right)^2}$ $k=\frac{5t}{1+t^2/a^2}$ $k=\frac{5a^2t}{a^2+t^2}$ $\frac{dk}{dt}=\frac{\left(a^2+t^2\right)-5a^2-5a^2t\left(2t\right)}{\left(a^2+t^2\right)^2}$ But at maximum concentration $\frac{dk}{dt}$=0 $5t^2a^2+5a^4-10a^2t^2=0$ At t=6 $180a^2+5a^4-360a^2=0$ $5a^2(a^2-36)=0$ Either 5a2=0 , a=0 Or a2-36=o, a= ± 6 A= ± 6		
(b) Volume of a cone. $\frac{1}{3}\pi r^2h]$ $v=\frac{1}{3\sqrt{3}}\pi r^3$ Given $\frac{dv}{dv}=\frac{\pi r^2}{\sqrt{3}}$ Required $\frac{dv}{dt}=\frac{dv}{dv}\,\frac{dv}{dt}$ $\frac{dr}{dt}=\frac{\sqrt{3}}{\pi}.\frac{dv}{t}$		

From $\frac{dv}{dt}$=9m/s When v=$\frac{6}{60}$ t=1 $v=\frac{6}{60}.20\sqrt{3}$ t=$20\sqrt{3}$ V=3$\sqrt{3}$ From V=$\frac{1}{3\sqrt{3}}\pi r^3$ $3\sqrt{3}=\frac{1}{3\sqrt{3}}\pi r^3$ $\pi r^3=27$ $r=\frac{3}{\pi^{\frac{1}{3}}}$ $\frac{dr}{dt}=\frac{\sqrt{3}}{\pi\left(\frac{3}{\pi^{1/3}}\right)^2}\frac{dv}{dt}$ $\frac{dr}{dt}=\frac{\sqrt{3}}{\pi^{1/3}}$		

P425/1
PURE
MATHEMATICS
Paper 1
Nov 2020
3hrs

ST. MARYS' KITENDE

Uganda Advanced Certificate of Education

RESOURCEFUL MOCK EXAMINATIONS 2020

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

*Attempt **all** the **eight** questions in **Section A** and**Not** more than **five** from **Section B.***

Any additional question(s) will not be marked.

All working must be shown clearly.

Silent non-programmabe calculators and mathematical tables with a list of formulae may be used.

Graph papers are provided.

SECTION A: (40MARKS)

*Answer **all** the **eight** questions in this Section.*

1. Solve the simultaneous equations; $\frac{1}{2y} + \frac{1}{x} = 4$; $\frac{3}{x} - \frac{1}{y} = 7$. (5marks)

2. Prove that; $\frac{log_2 x - log_2 x^2}{log_4 x^3} + \frac{5}{3} = log10$. (5marks)

3. Given the parabola $y^2 = 8x$,
a) Express a point T on the parabola in parametric form using t as the parameter. (2marks)
b) If parameter r gives point R, show that the gradient of chord TR is $\frac{2}{t+r}$. (3marks)

4. Find $\int x^3 e^{x^2} dx$. (5marks)

5. The line $r = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ a \\ b \end{pmatrix}$ meets a plane P perpendicularly at the point $(3, 1, 2)$.
Find the vector equation of the plane. (5marks)

6. Solve $\sin(120^0 + 3x) = cos(90^0 - x)\ for\ 0^0 \leq x \leq 90^0$. (5marks)

7. A roll of fencing material $152m$ long is used to enclose a rectangular area using two existing perpendicular walls. Find the maximum area enclosed. (5marks)

8. Solve the differential equation $\frac{dy}{dx}x - x = y$ given that $y = e$ when $x = e$. (5marks)

SECTION B : (60MARKS)

9. a) Prove that; ${}^{n+1}_{r+1}C + {}^{n+1}_{r+2}C = {}^{n+2}_{n-r}C$. (6marks)

b) Two blue, three red and four black beads are to be arranged on a circular ring made of a wire so that the red are separated. Find the number of different arrangements. (6marks)

10. Given that; $f(x) = \frac{1+2x}{1-x}$
a) Find Maclaurin's expansion of $f(x)$ upto the term in x^3. (8marks)
b) Hence, find the value of $\frac{1.02}{0.99}$ to four significant figures. (4marks)

11. a) Given that; $ysinx + xcosy = \frac{\pi}{2}$, find $\frac{dy}{dx}$. (4marks)

b) A square prism is always three times the width in length. If the volume increases at a constant rate of $4cm^3s^{-1}$, find the rate of change of the cross-sectional area when the width is $12cm$. (8marks)

12.

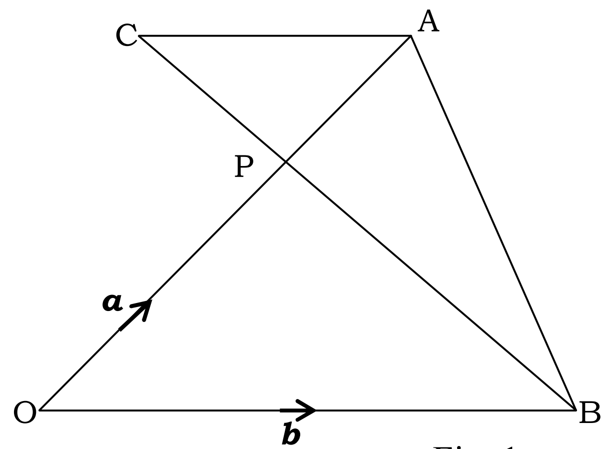


Fig. 1

Figure 1 shows points A and B with position vectors a and b respectively. $3AC = BO$.

a) Express each of the following in terms of vectors $a$and b.

i) BA (2marks)

ii) BC (3marks)

b) Find the ration $BP{:}PC$ (7marks)

13. a) Prove that $cos(tan^{-1}x) = (x^2 + 1)^{-\frac{1}{2}}$. (4marks)

b) i) Prove that $\frac{cos^2 4x + cos4x + sin^2 4x}{cos^2 4x - cos4x + si\ \ ^2 4x} = 3$ for $0 \leq x \leq \pi$. (4marks)

14. The lines L_1 and L_2 are perpendicular and intersect at $P(0,5)$. Line L_1 meets the x-axis in the first quadrant at Q such that $PQ = 13$ units. If L_2 meets the x-axis at R, without graphical construction, find the area of the triangle PQR. (12marks)

15. Given that $Z1 = 2 - 3i, Z_2 = 1 + 2i\ and\ Z_3 = 3 - 4i$.

a) Express $\frac{Z_1+Z_2}{Z_1Z_2}$ in the form $a + bi$ where a and b are real numbers. (6marks)

b) Find a polynomial $p(x)$ of degree four where the roots of $p(x) = 0$ are Z_2 and Z_3. (6marks)

16. Evaluate; $\int_2^3 \frac{x^4 - x^3 - x^2 + 4x - 1}{(x-1)(x^2+1)} dx$. (12marks)

END

P425/1
PURE MATHEMATICS
Paper 1
TUESDAY, 7th August 2018 (Morning)
3 hours

ACHOLI SECONDARY SCHOOLS EXAMINATIONS COMMITTEE

Uganda Advanced Certificate of Education

Joint Mock Examinations, 2018

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- ✓ Answer ALL the EIGHT questions in section A and any FIVE questions in section B.
- ✓ All necessary working MUST be shown clearly.
- ✓ Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 marks)

Answer ALL the questions in this section. All questions carry equal marks.

1. A curve is defined by the parametric equations: $x = t^2$, $y = \frac{1}{t}$ $(t \neq 0)$. Find the equation of the tangent to the curve at the point where the curve cuts the x-axis. (05 marks)

2. If $z = 2 + i$ is a root of the equation $2z^3 - 9z^2 + 14z - 5 = 0$, find the other roots. (05 marks)

3. (i) Find the binomial expression of $\frac{1}{(a + bx)^2}$ up to and including the term x^3.

(ii) Given that the coefficient of the x term is equal to the coefficient of the x^2 term, show that $3b + 2a = 0$. (05 marks)

4. Find the coordinates of the point C on the line joining the points A(–1, 2) and B(–9, 14) which divides AB internally in the ratio 1 : 3. Find also the equation of the line through C which is perpendicular to AB. (05 marks)

5. Solve the equation $5 \text{ Cos } \theta - 3 \text{ Sin } \theta = 4$ for $0^{\circ} \leq \theta \leq 360^{\circ}$. (05 marks)

6. Evaluate $\int_{2}^{5} x\sqrt{(x - 1)}\, dx$

(05 marks)

7. Find the Cartesian equation of the plane passing through the midpoint of AB with A(–1, 2, –5) and B(3, 0, –1) which is perpendicular to the line $\frac{x - 1}{2} = \frac{y + 7}{-3} = \frac{6 - z}{8}$. (05 marks)

8. Solve the equation: $\frac{dy}{dx} - y \tan x = \text{Cos } x$, given $y = 0$ at $x = \frac{\circledR}{2}$. (05 marks)

SECTION B (60 marks)

Answer only FIVE questions from this section. All questions carry equal marks.

Question 9:

(a)Evaluate $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\left(5x^3 - 12x + 4\right)}{\sqrt{\left(1 - x^2\right)}}\, dx$ (05 marks)

(b) By means of the substitution t = tan x, prove that $\int_0^{®/4} \frac{dx}{1 + \sin 2x} = \frac{1}{2}$ and find the value of $\int_0^{®/4} \frac{dx}{\left(1 + \sin 2x\right)^2}$. (07 marks)

Question 10:

(a) If z_1 and z_2 are complex numbers, solve the simultaneous equations:

$4z_1 + 3z_2 = 23$

$z_1 + iz_2 = 6 + 8i$, giving your answers in the form $x + iy$. (06 marks)

(b) Find the value of the complex number z given that $z^3 = \frac{5 + i}{2 + 3i}$. (06 marks)

Question 11:

(a) Find the angle between line $\frac{x - 2}{4} = \frac{y}{3} = \frac{z - 1}{2}$ and the plane $-3x + 5y + 6z = 10$. (04 marks)

(b) A plane P_1 passing through the points (1, –1, 0) and (1, 0, –3) is perpendicular to the plane P_2 having equation: $x + y = 6z = 0$. Find:

(i) the equation of P_1

(ii) the angle between P_1 and another plane P_3 with equation: $x - y + z = 7$.

(08 marks)

Question 12:

A disease is spreading at a rate proportional to the product of the number of people already infected and those who have not yet been infected. Assuming that the total number of people exposed to the disease is N;

(a) Write down a differential equation.

(b) Initially 20% of the population is infected. Two months later 40% of the population is infected. Determine how long it takes for only 25% of the population to remain uninfected. (12 marks)

Question 13:

(a) Determine the equation of the circle passing through the points A(–1, 2), B(2, 4) and C(0,4). (06 marks)

(b) If $y = mx - 5$ is a tangent to the circle $x^2 + y^2 = 9$, find the possible values of m. (06 marks)

Question 14:

Sketch the curve $y = \dfrac{x + 1}{(x - 1)(2x + 1)}$, showing clearly the asymptotes and turning points. (12 marks)

Question 15:

(a) Determine the maximum value of the expression: 6 Sin x – 3 Cos x. (03 marks)

(b) Prove that $\dfrac{\text{Cos}\,11^\circ + \text{Sin}\,11^\circ}{\text{Cos}\,11^\circ - \text{Sin}\,11^\circ} = \tan 56^\circ$ (03 marks)

(c) In a triangle ABC, prove that $\text{Sin}\,B + \text{Sin}\,C - \text{Sin}\,A = 4\,\text{Cos}\,\dfrac{A}{2}\,\text{Sin}\,\dfrac{B}{2}\,\text{Sin}\,\dfrac{C}{2}$ (06 marks)

Question 16:

(a) Using Maclaurin's theorem, expand e^{-x}Sin x up to the term in x^3. Hence, evaluate $e^{-®/3}\,\text{Sin}\,\dfrac{®}{3}$ to 4 significant figures. (05 marks)

(b) The curve $y = x^3 + 8$ cuts the x and y axes at the points A and B respectively. The line AB meets the curve again at point C. Find the coordinates of A, B, C and hence, find the area enclosed between the curve and the line. (07 marks)

THE END

P425/1
PURE MATHEMATICS
Paper 1
July/Aug. 2020
3 hours

ASK INTERGRATED TEACHERS' MOCK EXAMINATIONS BURREAU

AITEL JOINT MOCK EXAMINATIONS

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES

Answer **all** *the* **eight** *questions in section* **A** *and any* **five** *questions from section* **B**

Any additional question(s) answered will **not** *be marked*

Show **all** *the necessary workings clearly*

Begin each question on a fresh page of paper

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used

 Turn Over

SECTION A (40 MARKS)

Attempt **all** *questions in this section*

1. Given that $a = \log_5 35$ and $b = \log_9 35$, show that $\log_5 21 = \frac{1}{2b}(2ab - 2b + a)$.

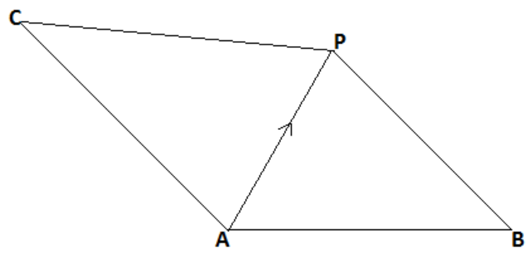
(05 marks)

2. Solve the inequality: $|x-2| > 3|2x+1|$. (05 marks)

3. Prove that: $\tan^{-1}\frac{1}{2} - \operatorname{cosec}^{-1}\frac{\sqrt{5}}{2} = \cos^{-1}\frac{4}{5}$. (05 marks)

4. Expand $\frac{1}{\sqrt{1+x}}$ up to the term in x^2 and by letting $x = \frac{1}{4}$, show that $\sqrt{5} \approx \frac{256}{115}$. (05 marks)

5.



$A,\ B,\ C$ and P are four points such that $3\overrightarrow{\mathbf{AP}} = 2\overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{AC}}$, show that $B,\ P$ and C are collinear and that P is the point of trisection of the line BC.

(05 marks)

6. Given that $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$, show that $\frac{d^2y}{dx^2} + 2y\frac{dy}{dx} = 0$. (05 marks)

7. Find the volume of the solid generated by rotating the area bounded by the curve $y = \cos\frac{1}{2}x$ from $x = 0$ to $x = \pi$ about the $x-$ axis. (05 marks)

8. Solve the d.e given $\cos x\frac{dy}{dx} - 2y\sin x = 1$. (05 marks)

SECTION B (60 MARKS)

Attempt any **five** *questions in this section*

9 (a) Evaluate the coefficient of x in the expansion of $\left(x+\frac{2}{x^2}\right)^{10}$. (05 marks)

(b) Prove by Mathematical induction that: $\sum_{r=2}^{n}\frac{1}{r^2-1}=\frac{3}{4}-\frac{2n+1}{2n(n+1)}$. (07 marks)

10 (a) Prove the identity: $\cos^6 x+\sin^6 x=1-\frac{3}{4}\sin^2 2x$. (06 marks)

(b) Solve the equation: $4\sin^2 x+8\cot^2 x=5\operatorname{cosec}^2 x$ for $0\le x\le 2\pi$. (06 marks)

11. Sketch the curve $y=\frac{4+3x-x^2}{x-8}$, clearly find the nature of the turning points and state their asymptotes. (12 marks)

12 (a) The points $P\left(5p,\ \frac{5}{p}\right)$ and $Q\left(5q,\ \frac{5}{q}\right)$ lie on the rectangular hyperbola $xy=25$. Find the equation of the tangent at P and hence deduce the equation of the tangent at Q. (05 marks)

(b) The tangents at P and Q meet at point N, show that the coordinates of N are $\left(\frac{10pq}{p+q},\ \frac{10}{p+q}\right)$, hence find the locus of N given that $pq=-1$. (07 marks)

13 (a) Given that $z(5+5i)=a(1+3i)+b(2-i)$ where a and b are real numbers and that $\arg z=\frac{\pi}{2}$ and $|z|=7$, find the values of a and b. (06 marks)

(b) Describe the locus of the complex number z when it moves in the argand diagram such that $\arg\left(\frac{z-3}{z-2i}\right)=\frac{\pi}{4}$. (06 marks)

14 (a) Evaluate: $\int_0^{\pi/3} x\sin 3x\,dx$ (06 marks)

Turn Over

(b) Prove that: $\int_{\pi}^{4\pi/3} \operatorname{cosec} \frac{1}{2} x \, dx = In3$ (06 marks)

15 (a) Find the point of intersection between the plane $\mathbf{r} \cdot (\mathbf{2i} - \mathbf{j} + \mathbf{k}) = \mathbf{4}$ and the line passing through the point $(3, 1, 2)$ and is perpendicular to this plane.

(05 marks)

(b) Find the perpendicular distance of the point $(4, -3, 10)$ to the line

$\frac{x-1}{3} = 2 - y = \frac{z-3}{2}$. (07 marks)

16. A liquid is being heated in an oven maintained at a constant temperature of $180^o C$. It is assumed that the rate of increase in the temperature of the liquid is proportional to $(180 - \theta)$, where $\theta^o C$ is the temperature of the liquid at time t minutes. If the temperature of the liquid rises from $0^o C$ to $120^o C$ in 5 minutes, find the temperature of the liquid after a further 5 minutes. (12marks)

END

P425/1
PURE
MATHEMATICS
Paper 1
July/Aug. 2022
3 hours

AITEL JOINT MOCK EXAMINATIONS

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 Hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in the section* **A** *and*

Answer any **five** *questions in section* **B**

 Turn Over

SECTION A (40 MARKS)

1. The roots of a quadratic equation $x^2+(7+p)x+p=0$ are ∝ and β.
Given that ∝and β differ by 5, find the possible values of p. (5 marks)

2. Evaluate $\int_2^3 \frac{2}{x^2-4x+13}dx$ (5 marks)

3. Show that the parametric equations $x=5+\frac{\sqrt{3}}{2}\cos\theta \quad y=-3+\frac{\sqrt{3}}{2}\sin\theta$ represent a circle. Find the radius and centre of the circle. (5 marks)

4. Solve the equation: $3+2\cos^2\theta+\tan\theta=4\cos^2\theta$, for $0^0 \leq \theta \leq 360^0$ (5 marks)

5. How many team of 6 players can be formed from a group of 7 boys and 5 girls if
   (i) Each team should have at least 3 boys and a girl
   (ii) Each team contains at most 3 girls (5 marks)

6. Use raw Echelon reduction to solve the following Equations simultaneously.
   (i) $3x=z-2y$
   (ii) $3y=x+2z+1$
   (iii) $3z=2x-2y+3$ (5 marks)

7. Differentiate $y=x\ln x$ from first principles . (5 marks)

8. Find the equation of the normal to the curve $x^2\tan x-xy-2y^2=-2$ at the point (0,1) (5 marks)

SECTION B (60 MARKS)

Answer any **five** *questions*

9. (a) Show that the lines $r = 5+3-5k + u\,(+2j - 3k)$ and $\frac{x-7}{3}=\frac{y+1}{-2}=\frac{z+4}{-2}$ intersect and hence find the coordinates of the point of intersection. (7 marks)

   (b) A plane is at a distance of $\sqrt{11}$ units from the origin. If theline passing through the points A(4,-9,3) and B(6,-7,9) is perpendicular to the plane, find the cartesian equation of the plane. (5 marks)

10. (a) Show that sin3θ= 3sinθ – 4sin$^3\theta$ hence solve the equation $8x^3-6x-1=0$ (7 marks)

    (b) By expressing 5cosx + 8sinx in the form $Rcos(x+\beta)$, where R is a constant and β is an acute angle, solve $5cosx + 8sinx = 7$ for $0^0 \leq$ x $\leq 360^0$ (5 marks)

11. Evaluate $\int_{2}^{3} \frac{2x+1}{(x-2)(x+1)^2} dx$. Give your answer correct to 3 decimal places.

(12 marks)

12. (a) Prove by induction that $4^{(n+3)} - 3n - 10$ is divisible by 3 for all positive integral values of *n*. (6 marks)

(b) Use De Moivres theorem to prove that the complex number $(\sqrt{3}+i)^n + (\sqrt{3}-i)^n$ is always real and hence find the value of the expression when n = 6. (6 marks)

13. (a) Use the binomial theorem to show that

$$(\sqrt{1+2x} + \sqrt{1-4x})^2 = (2 - x - \frac{5}{2}x^2 + ...)^2$$ (7 marks)

(b) Taking $x = \frac{1}{16}$ use the equation in (a) above to estimate $\sqrt{6}$ to 2 decimal places.

(5 marks)

14. (a) Solve for x in; $9^x - 3^{(x+1)} = 10$ (5 marks)

(b) Solve the following pair of simultaneous equations.

$$\log_2 x^2 + \log_2 y^3 = 1$$
$$\log_2 x - \log_2 y^2 = 4$$
(7 marks)

15. (i) If the curve $y = \frac{x^2 - 4x + 4}{x+1}$ Show that the curve is restricted, state the region and hence investigate the nature of the turning points.

(ii) Determine the equations of asymptotes to the curve

(iii) Sketch the curve

(12 marks)

16. (a) Solve the differential equation.

$y\cos^2 x \frac{dy}{dx} = \tan x + 2$, given that y = 0 when $x = \frac{\pi}{4}$ (5 marks)

(b) A spherical bubble evaporates at a rate proportional to its surface area. If half of it evaporates in 2 hours, when will the bubble disappear? (7 marks)

END

P425/I
PURE MATHEMATICS
PAPER 1
JUL/AUG 2022
3HOURS

ST.MARY'S COLLEGE – KITENDE

INTERNAL ASSESSMENT EXAMINATIONS
Uganda advanced certificate of education
PURE MATHEMATICS
PAPER 1
3HOURS

Instructions

- Attempt all the eight questions from section A and only five questions from section B.
- All your working must clearly be shown.
- Begin each answer on a fresh sheet of paper.
- Mathematical tables and graph papers are provided.
- Silent non-programmable calculators may be used.

SECTION A (40 marks)

Answer all the questions from this section.

1. If X is sufficiently small enough to allow any terms in x^5 or higher powers of x to be neglected, show that $1+x)^6 \ (1-2x^3)^{10} = 1 + 6x + 15x^2 - 105x^4$. (05 marks)

2. A straight line AB of length 10 units is free to move with its ends on the axes. Find the locus of a point P on the line at a distance of 3 units from the end on the axes.

   (05 marks)

3. Find $\int \frac{ln\,(1+ln\,x)dx}{x}$

   (05 marks)

4. Find the perpendicular distance from the point P (4, 6, 4) to the line passing through the points A (2, 2, 1) and B (4, 3, -1).

   (05 marks)

5. Given that y = e^{2x}cos3x, show that $\frac{d2y}{dx}$ - $4\frac{dy}{dx}$ + 13y = 0

   (05 marks)

6. Solve the equation $2\tan\theta + \sin2\theta\sec\theta = 1 + \sec\theta$ in the range $0^0 \leq \theta \leq 360^0$.

   (05 marks)

7. Prove by induction that $2^{4n} - 1$ is a multiple of 15 if n is a natural number.

   (05 marks)

8. Water is poured into a vessel in the shape of a right circular cone of vertical angle 90^0 with the axis vertical, at the rate of $125cm^3s^{-1}$. At

what rate is the level of water surface rising when the depth of the water is 10cm? (05 marks)

SECTION B (60 marks)

Answer only five questions from this section.

9. (a) Given Z = -10 + 9i as a complex number

(i) Find the complex number w which satisfy the equation zw = -11 + 28i (04 marks)

(ii) Verify that $|z + w| = \sqrt[8]{2}$.

(02 marks)

(b) Express that $\sqrt{3} + i$ in the modulus argument form. Hence find $(\sqrt{3} + i)^{10}$ in the form a + bi.

(06 marks)

10. (a) Evaluate $\int_0^{\frac{\pi}{3}} (1 + \cos 3x)^2 \, dx$

(05 marks)

(b) Use the substitution $t = \tan\frac{x}{2}$ and find $\int \frac{\cos x}{1-\cos x} dx$

(07 marks)

11. (a) Given that $x^3 + 5x^2 + ax + b$ is divisible by $x^2 + x - 2$, find

(i) the values of a and b.

(05 marks)

(ii) the linear factor of the polynomial .

(02 marks)

(b) Find the number of arrangement of all the letters of the word MATHEMATICS in a row.

(i) without restriction

(02 marks)

(ii) in which the A's are separated .

(03 marks)

12. (a) Prove that in any triangle ABC $\frac{bc}{ab+ac} = \frac{\operatorname{cosec}(B+C)}{\operatorname{cosec}B+\operatorname{cosec}C}$

(06 marks)

(b) Solve the equation $\tan^{-1}(2x+1) + \tan^{-1}(2-1) = \tan^{-1} 2$

(06 marks)

13. (a) Differentiate $\frac{\sin x}{x^2+\cos x}$ with respect to x.

(b) Express $f(x) = \frac{1}{(x+2)(1+x)^2}$ into partial fractions and hence find the definite integral of $\int_0^1 f(x)dx$.

(08 marks)

14. The co-rdinates of the points A and B are (0,2,5) and (-1, 3, 1) respectively

and the equation of the line L is $\frac{x-3}{2} = \frac{y-2}{-2} = \frac{z-2}{-1}$

(i) Find the equation of the plane containing the point A and perpendicular to L and verify that B lies in the plane.

(06 marks)

(ii) Find the position vector of the point of intersection of the line L and the plane in (i) above.

(06 marks)

15. A tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point P(a $\cos\theta$, b$\sin\theta$) meets the minor axis at L. If the normal at P meets major axis t=at M. Find the locus of the midpoint of LM.

(12 marks)

16. In a certain type of chemical reaction a substance is continuously transformed

into a substance B throughout the reaction, the sum of the masses of A and B remains constant and equal to M. The mass of B present at time t after the commencement of the reaction is donated by x. At any time, the rate of increase of mass of A where k is constant.

(a) Write down a differential equation relating x and t.

(b) Solve this differential equation given that x= 0 and t= 0. Given also that x $=\frac{1}{2}$m where t = In 2, determine the value of k and show at time t x = m$(1 - e^{-t}$.Hence find

(i) the value of x interms of m when t = 3In2.

(ii) the value of t when x = $\frac{3}{4}$m.

(12marks)

THE END

P425/2
APPLIED MATHEMATICS
3 hours

UGANDA ADVANCED CERTIFICATE OF EDUCATION - 2022

INSTRUCTIONS TO CANDIDATES:

- *Answer **all** the **eight** questions in Section **A** and any **five** from Section **B**.*
- *Any additional questions answered will **not** be marked.*
- ***All** working **must** be shown clearly.*
- *Graph paper is provided.*
- *Silent non-programmable **scientific calculators** and **mathematical tables** with a list of formulae may be used.*
- *In numerical work, take g to be 9.8ms^{-2}*

SECTION: A (40marks)

1. From the top of a building 45m high, a stone is projected upwards with a speed Vms^{-1} at an angle of 30^0 to the horizontal. Two seconds later another stone is dropped from the same point. If the stones reach the ground at the same time. Find the value of V. (5marks)

2. Events A and B are such that $P\left(A/B\right)=\frac{2}{3}, P\left(A/B^1\right)=\frac{2}{5}$ and $P(B)=\frac{4}{7}$. Find the;
   (i) $P(A)$ (3marks)
   (ii) $P\left(B/A^1\right)$ (2marks)

3. The table below shows values of $\tan\theta$.

θ	1.11	1.15	1.19	1.23
$\tan\theta$	2.0143	2.2345	2.4979	2.8198

   Use linear interpolation or extrapolation to find;
   (i) $\tan\theta$ when $\theta = 1.17$.
   (ii) θ when $\tan\theta = 2.923$ (5marks)

4. A body of mass 5kg initially at rest at a point with position vector $(2i - j)$m is acted upon by a force $(2i - 4j - 3k)N$. Find the;
   (i) Position of the body,
   (ii) Work done by the force after the 2 seconds. (5marks)

5. The table below shows the expenditure of a certain family for months September and October in 2020.

ITEMS	EXPENDITURE(Shs.)		WEIGHT
	SEPTEMBER	OCTOBER	
Food	300,000	325,000	5
Accomodation	260,000	365,500	3
Electricity	150,000	160,000	1
Miscellaneous	620,000	725,000	2

Calculate the cost of living index for the month of October based on september in 2020. (5marks)

6. Use the trapezium rule with 6 ordinates to evaluate $\int_1^{1.2} x^2 \sin\left(\frac{1}{2}x\right)dx$ giving your answer correct to three decimal places. (5marks)

7. The distribution function of a continuous random variable X is as follows:

$$F(x) = \begin{cases} 0 & , \quad x < 1 \\ \frac{1}{4}(x-1)^2 & , \quad 1 \le x \le 3 \\ 1 & , \quad x > 3 \end{cases}$$

Find: (i) $P(1.5 \le X \le 2)$. (3marks)

(ii) The p.d.f of X. (2marks)

8. A non-uniform ladder AB of weight 78.4N ad length 5m is freely suspended horizontally by two light inelastic strings AC and BD that make angles 30^0 and 40^0 respectively with the vertical; find the distance from A, where weight of the ladder acts. (5marks)

SECTION:B (60marks)

9. (a)The weight of a particular variety of mangoes is normally distributed with mean 205grams and standard deviation 25grams .Find the probability that a mango chosen at random from the variety is ;

(i) less than 250grams

(ii) between 200grams and 250grams. (6marks) from the instant they are called is approximately normally distributed. the value of X was recorded on a random sample of 50 Occassions on which the fire brigade was called and the results summarised below.

$\sum x = 286.5$, $\sum (x-\bar{x})^2 = 45.16$

Determine the 98.5% confidence interval for the mean time taken by the town fire brigade team to reach a fire scene for all the occasions in town from the instant they are called. (6marks)

10. (a) The numbers X and Y were estimated with maximum errors of ΔX and ΔY respectively. Show that the maximum possible relative error in the estimation of X^2Y is given by $2\left|\frac{\Delta X}{X}\right| + \left|\frac{\Delta y}{Y}\right|$. (5marks)

(b) Given that the numbers A= 7.4, B= 5.42 and C= 9.80 are rounded off with percentage errors 2, 3 and 1 respectively, calculate the relative error made in evaluating $\frac{B}{A-C}$, correct to two decimal places. (7 marks

11. (a)To an observer on a train travelling at 3kmh^{-1},a bird appears to fly due west at 4 kmh^{-1}.If the bird actually travels due North-West, find its speed. (5marks)

(b)At time $t = 0$, particles A and B are moving with constant velocities $(\mu i + 3j + 30k)ms^{-1}$ and $(4i - 2j - 15k)ms^{-1}$ are located at position vectors $(2i + j - 15k)m$ and $(-i + 4j + 12k)m$ respectively. Find:

(i) value of μ such that A and B will collide,

(ii)the value of t when this collision occurs (7marks)

12. The table below shows marks obtained by 8 candidates in physics and mathematics.

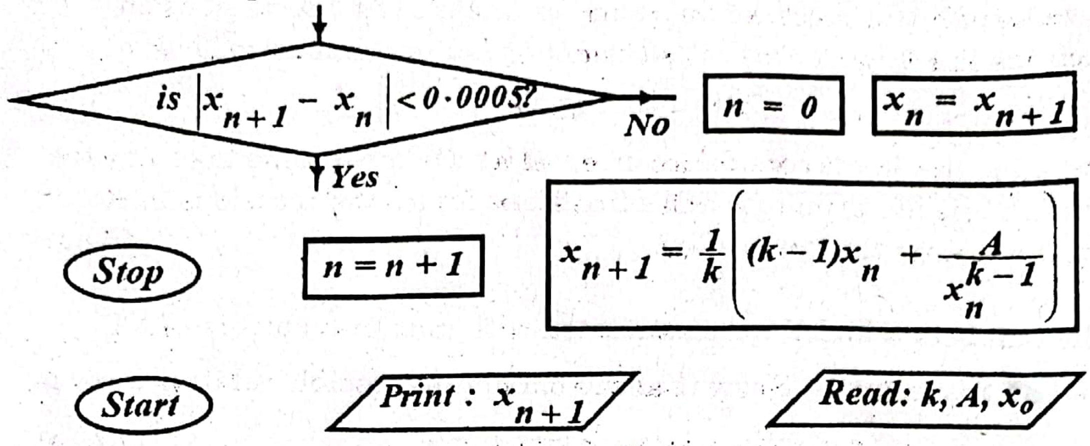
Candidate	A	B	C	D	E	F	G	H
Mathematics (X)	52	65	41	65	81	31	65	55
Physics (Y)	50	60	35	65	66	35	69	48

(a) calculate the rank correlation coefficient for the data and comment on the significance of mathematics on physics at 1% level. (5marks)

(b) (i) plot a scatter diagram for the scores in mathematics and physics

(ii) Draw a line of best fit hence find the marks scored in physics by a student who scored 75 marks in mathematics (7marks)

13. Given below are parts of the flow chart not arranged in order.



i) By re-arranging the given parts, draw a logical flow chart.

ii) Using $x_0 = 1.6, A = 28$ and $k = 6$, perform a dry run for the flow chart.

iii) State the purpose of the flow chart, basing on the values given in (ii) above. (12marks)

14. A body of mass $m$kg lies on a rough plane inclined at θ^0 to the horizontal. When a force of $\frac{mg}{2}N$ parallel to and up the plane is applied to the body, it is just about to move up the plane. When a force of $\frac{mg}{4}N$ parallel to and down the plane is applied to the body, it just about to move down the plane.

Calculate the;

i) Value of θ.

ii) Coefficient of friction between the body and the plane. (12marks)

15. (a)In a certain school 15% of the students are left-handed. Determine the probability that in a random sample of 10 students ;

(i) exactly 3 are left-handed, (2marks)

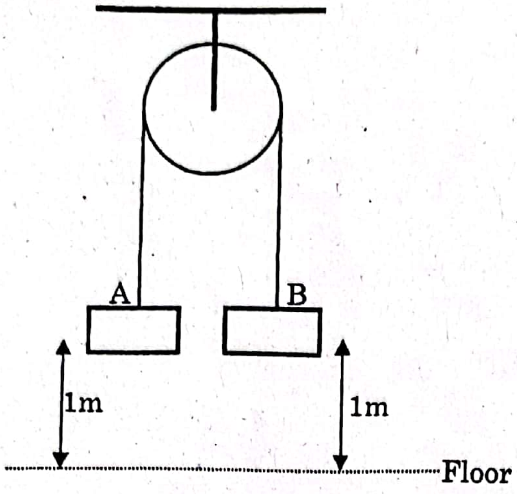
(ii) at least 8 are left-handed (3marks)

(b)Tommy and her daughter Lisa, support their town soccer team. When their town soccer team plays, the probability that Tommy watches the game is 0.8.The probability that Lisa watches the game when her father watches the game 0.9 otherwise it is 0.4. Calculate the probability that;

(i)Neither Tommy nor Lisa watches a particular game (3marks)

(ii)Tommy watches a particular game when Lisa does not watch the game (4marks)

16.



Two particles A and B have mass0.4kg and 0.3kg respectively. The particles are attached to ends of a light inextensible string. The string passes over a light smooth pulley which is fixed above a horizontal floor. Both particles are held at rest, with the string taut, at a height 1m from above the floor as shown in the diagram above. The particles are released from rest and in the subsequent motion B does not reach the pulley.

(a) Find the tension in the string and the acceleration of the particles immediately after they are released. (6marks)

(b)when the particles have been moving for 0.5 seconds, the string breaks. Find the further time that elapses until B hits the floor. (6marks)

END

SECTION A: (40 MARKS)

Answer **all** *the questions in this section.*

1. A force $(3i - 2j + 8k)$ N acts on a body of mass 4kg initially at the origin. If the velocity is $(2ti + 3j)$ ms^{-1}, find the work done after 4 seconds.

(05 marks)

2. The temperature (°C) of a liquid measured at an interval of 2 minutes were recorded as 55 and 52. If the initial temperature is 60, use linear interpolation or extrapolation to find;
   (i) Temperature after 5 minutes,
   (ii) Time taken if the temperature is 53.5°C.

3. A random sample of 200 people were asked the length of time they spent in the shower, the last time they took one. The results were as follows:
   $\sum x = 909$, $\sum x^2 = 4555$.
   (a) Calculate the unbiased estimate of the population variance. *(02 marks)*
   (b) Determine the 97.5% confidence limits for the mean time spent in the shower. *(03 marks)*

4. To a dove flying eastwards at $3ms^{-1}$ an eagle appears to be flying North East wards at $4ms^{-1}$. Find the true velocity of the eagle. *(05 marks)*

5. The numbers $x = 4.2$, $y = 16.02$ and $z = 25$ are rounded off with corresponding percentage errors of 0.5, 0.45 and 0.02, Calculate the absolute error made in $\frac{xy}{z}$.

(05 marks)

6. A and B are two independent events with A twice as likely to occur as B. if $P(A) = \frac{1}{2}$, find:

(i) P(A or B but not both), *(03 marks)*

(ii) $P(A/B')$. *(02 marks)*

7. Forces of magnitude 90N and 60N act on a particle at angle of 35^0 to each other. Determine the magnitude and direction of the resultant force.

(05 marks)

8. The probability that a certain function starts early is $^4/_7$.
If the function starts early, the probability that it takes a longer time is $^2/_5$.
If the function starts late, the probability that it takes a shorter time is $^1/_3$. Find the probability that function;

(i) Takes a shorter time, *(03 marks)*

(ii) Starts early if it takes a shorter time *(02 marks)*

SECTION B: (60 MARKS)

*Answer any **five** questions from this section.*

All *questions carry equal marks*

9. The table below shows the marks obtained by students in Fine Art(x) and mathematics (y).

Fine Art (x)	80	76	96	41	68	31	42	88	68	91
Mathematics (y)	43	32	27	64	65	64	65	32	64	43

(a) Draw a scatter diagram for the above data and on it draw a line of best fit. Use the line of best fit to estimate the mark of a student who scored;

(i) 61 in mathematics,

(ii) 25 in Fine Art. *(07 marks)*

(b) Calculate a rank correlation coefficient between the students' performance in the two subjects and comment on your result at 1% level of significance. *(05 marks)*

10. A particle is projected with a speed of $36ms^{-1}$ at an angle of 40^0 to the horizontal from a point 0.5m above the level ground. It just clears a wall which is 70 meters on the horizontal plane from the point of projection. Find the;

(a) (i) time taken for the particle to reach the wall.

(ii) height of the wall. *(08 marks)*

(b) Maximum height reached by the particle from the point of projection. *(04 marks)*

11. (a) Use the trapezium rule with six ordinates to find the approximate value of $\int_{0.5}^{1.5}\left(\frac{3}{x}+x^4\right)dx$, correct to three significant figures, (05 marks)

(b) Evaluate $\int_{0.5}^{1.5}\left(\frac{3}{x}+x^4\right)dx$ correct to three significant figures.

(c) (i) Determine the percentage error in the estimation in (a) above, correct to two decimal places. *(03 marks)*

(ii) Suggest how the percentage error may be reduced. *(01 mark)*

12. The continuous random variable X has probability density function (p.d.f)

given by; $f(x) = \begin{cases} (4x - 4x^3); & 0 \leq x \leq 1 \\ 0 & ; otherwise \end{cases}$

Find the

(a) Mode *(03 marks)*

(b) Cumulative distribution function of x, *(03 marks)*

(c) P $(0.1 < x < 0.6)$ *(02 marks)*

(d) Median of x *(04 marks)*

13. Forces of magnitude 4N, 5N, 5N, 4N and 6N act along the lines **AB, BC, CD, DA** and **AC** respectively of the square **ABCD** whose side has a length of a units. The direction of the forces are indicated by the order of the letters.

(a) Find the magnitude and direction of the resultant force. (09 marks)

(b) If the line of action of the resultant force cuts **AB** produced at E, find the length **AE.** *(03 marks)*

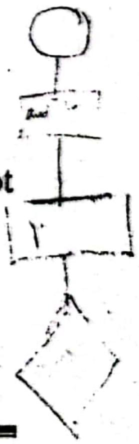
14. (a) Derive the simplest formula based on Newton Raphson's method to show that for the equation $3x = ln3$ it satisfies

$$x_{r+1} = \frac{1}{3}\left\{\frac{e^{3x_r}(3x_r - 1) + 3}{e^{3x_r}}\right\}$$ *(04 marks)*

(b) (i) Construct a flow chart that;

- reads the initial approximation as x_0
- computes, using the iterative formula in (a), and prints the root of the equations $3x = in3, to$ 4 significant figures.

(ii) Perform a dry-run for your flow chart for $x_0 = {}^1/_3$ *(08 marks)*



P425/1
Pure Mathematics
Paper 1
July - August, 2022
3 hours

UGANDA MUSLIM TEACHERS' ASSOCIATION

UMTA JOINT MOCK EXAMINATIONS 2022

UGANDA ADVANCED CERTIFICATE OF EDUCATION

Pure Mathematics

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES

- *Attempt **all** the **eight** questions in section A and **five** questions from section **B**.*
- *Any additional question(s) answered will not be marked*
- ***All** working **must** be shown clearly. Begin each question on a fresh sheet of paper.*
- *Silent, nonprogrammable scientific calculators and mathematical tables with a list of formulae may be used.*

SECTION A

1. If $y = \tan^{-1}\left(\frac{ax-b}{bx+a}\right)$ show that $\frac{dy}{dx} = \frac{1}{1+x^2}$ (5 marks)

2. Evaluate $\int_1^4 \frac{x^2+x}{\sqrt{2x+1}}dx$ (5 marks)

3. $\frac{Sinx + Sin3x + \sin 5x}{Cosx + Cos3x + \cos 5x} = \tan 3x$ (5 marks)

4. Find the angle between x - 3y +5 = 0 and x +2y -1 = 0 (5 marks)

5. Find the area enclosed by the curve $y = x - \frac{1}{x}$, the x-axis and line x = 2 (5 marks)

6. Solve the equation $\sqrt{(3x-x)} - \sqrt{(7+x)} = \sqrt{16+2x}$ (5 marks)

7. Calculate the number of different 7-arrangements which can be made with the letters of the word MAXIMUM. In how many of these do the 4 consonants all appear next to one another? (5 marks)

8. Find the length of the perpendicular distance from A(4,3,5) to plane 6x-y +2z = 14

   (5 marks)

SECTION B

9. (a) Find the foot of the perpendicular drawn from the point (2, -1, 5) to the line $\frac{x-11}{10} = \frac{y+2}{-2} = \frac{z+5}{-11}$ (6 marks)

   (b) Find the angle between the plane x-2y + z =20 and line $\frac{2-x}{-3} = \frac{y+1}{4} = \frac{2-z}{-12}$

   (6 marks)

10. (a) A conical vessel whose height is 10meters and the radius of the base 5m is being filled with water at a uniform rate of $1.5m3min^{-1}$. Find the rate at which the level of the water in the vessel is rising when the depth is 4 meter.

    (6 marks)

    (b) Find the area enclosed between the curve y= x(x-1) (x-2) from x = 0 to x = 2

    (6 marks)

11. Partialise fully $f(x)=\frac{x^4+x^3-6x^2-13x-6}{x^3-7x-6}$. Hence $\int f(x)dx$ from 4 to 5

(12 marks)

12. (a) Solve the differential equation

$$\frac{dy}{dx}+\frac{2xy}{x^2+1}-x=0$$

(4 marks)

(b)A liquid cools in a room at a constant temperature of 22^0c at a rate proportional to the excess temperature. Initially the temperature of the liquid was 100^0C and 1 minute later it was 92.2^0C. Find the temperature of the liquid after 5 minutes. (8 marks)

13. (a)Solve the equation $2^{2x+8}-32(2^x)+1=0$ (5 marks)

(b) If $x = log_a bc$ $y = log_b ac$ and $z = log_c ab$ Prove that $x + y + z = xyz - 2$ (8 marks)

14. (a) Expand $(1-3x)^{1/3}$ in ascending powers of x up to x^4. By using $x=\frac{1}{8}$

evaluate $5^{\frac{1}{3}}$. Give your answer to two decimal places. (6 marks)

(b) Determine the two ranges of real values of x which satisfy the inequality

$$\frac{x-2}{x-1}\leq\frac{x+2}{x+1}$$

(6 marks)

15. (a) Find the values of x that satisfy the equation (6 marks)

$10\sin^2 x + 10\sin x\sin x - \cos^2 x = 2$ between 0^0 and 360^0

(b) Show that $Cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{27}{11}\right)$

(6 marks)

16. (a) Given that one root of the equation $Z^4-6Z^3+23Z^2-34Z+26=0$ is $1+i$ find the others. (6 marks)

(b) If **Z** is a general complex number on argand diagram. Show the region given by

$|z+1-4i|>|z-2-i|$ (6 marks)

END

P425/1
Pure Mathematics
Paper 1
July - August, 2022
3 hours

UGANDA MUSLIM TEACHERS' ASSOCIATION

UMTA JOINT MOCK EXAMINATIONS 2022

UGANDA ADVANCED CERTIFICATE OF EDUCATION

Pure Mathematics

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES

- *Attempt **all** the **eight** questions in section A and **five** questions from section **B**.*
- *Any additional question(s) answered will not be marked*
- ***All** working **must** be shown clearly. Begin each question on a fresh sheet of paper.*
- *Silent, nonprogrammable scientific calculators and mathematical tables with a list of formulae may be used.*

SECTION A

1. If $y = \tan^{-1}\left(\frac{ax-b}{bx+a}\right)$ show that $\frac{dy}{dx} = \frac{1}{1+x^2}$ (5 marks)

2. Evaluate $\int_1^4 \frac{x^2+x}{\sqrt{2x+1}}dx$ (5 marks)

3. $\frac{Sinx + Sin3x + \sin 5x}{Cosx + Cos3x + \cos 5x} = \tan 3x$ (5 marks)

4. Find the angle between x - 3y +5 = 0 and x +2y -1 = 0 (5 marks)

5. Find the area enclosed by the curve $y = x - \frac{1}{x}$, the x-axis and line x = 2 (5 marks)

6. Solve the equation $\sqrt{(3x-x)} - \sqrt{(7+x)} = \sqrt{16+2x}$ (5 marks)

7. Calculate the number of different 7-arrangements which can be made with the letters of the word MAXIMUM. In how many of these do the 4 consonants all appear next to one another? (5 marks)

8. Find the length of the perpendicular distance from A(4,3,5) to plane 6x-y +2z = 14

(5 marks)

SECTION B

9. (a) Find the foot of the perpendicular drawn from the point (2, -1, 5) to the line $\frac{x-11}{10} = \frac{y+2}{-2} = \frac{z+5}{-11}$ (6 marks)

   (b) Find the angle between the plane x-2y + z =20 and line $\frac{2-x}{-3} = \frac{y+1}{4} = \frac{2-z}{-12}$

   (6 marks)

10. (a) A conical vessel whose height is 10meters and the radius of the base 5m is being filled with water at a uniform rate of 1.5m3min^{-1}. Find the rate at which the level of the water in the vessel is rising when the depth is 4 meter.

    (6 marks)

    (b) Find the area enclosed between the curve y= x(x-1) (x-2) from x = 0 to x = 2

    (6 marks)

11. Partialise fully $f(x)=\frac{x^4+x^3-6x^2-13x-6}{x^3-7x-6}$. Hence $\int f(x)dx$ from 4 to 5

(12 marks)

12. (a) Solve the differential equation

$$\frac{dy}{dx}+\frac{2xy}{x^2+1}-x=0$$

(4 marks)

(b)A liquid cools in a room at a constant temperature of 22^0c at a rate proportional to the excess temperature. Initially the temperature of the liquid was 100^0C and 1 minute later it was 92.2^0C. Find the temperature of the liquid after 5 minutes. (8 marks)

13. (a)Solve the equation $2^{2x+8}-32(2^x)+1=0$ (5 marks)

(b) If $x = log_abc$ $y = log_bac$ and $z= log_cab$ Prove that $x + y + z = xyz - 2$ (8 marks)

14. (a) Expand $(1-3x)^{1/3}$ in ascending powers of x up to x^4. By using $x=\frac{1}{8}$

evaluate $5^{\frac{1}{3}}$. Give your answer to two decimal places. (6 marks)

(b) Determine the two ranges of real values of x which satisfy the inequality

$$\frac{x-2}{x-1}\le\frac{x+2}{x+1}$$

(6 marks)

15. (a) Find the values of x that satisfy the equation (6 marks)

$10\sin^2x + 10\sin x\sin x - \cos^2x = 2$ between 0^0 and 360^0

(b) Show that $Cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{27}{11}\right)$

(6 marks)

16. (a) Given that one root of the equation $Z^4 - 6Z^3 + 23Z^2 - 34Z + 26 = 0$ is $1+i$ find the others. (6 marks)

(b) If **Z** is a general complex number on argand diagram. Show the region given by

$|z+1-4i|>|z-2-i|$ (6 marks)

END